Credit Cards, the Demand for Money, and Monetary Aggregation*

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We use nonparametric and parametric demand analysis to empirically estimate a credit card-augmented monetary asset demand system, based on the Minflex Laurent flexible functional form, and a sample period that includes the 2007-2009 global financial crisis and the Covid-19 pandemic. We also use multivariate copulae in an attempt to capture various patterns of dependence structures. In doing so, we relax the joint normality assumption of the errors of the demand system and estimate the model without having to delete one equation as is usually the practice. We show that the Minflex Laurent copula-based demand system produces a higher income elasticity for credit card transaction services and higher Morishima elasticities between credit card transaction services and monetary assets compared to the traditional estimation of the Minflex Laurent demand system. We also show that credit cards are substitutes for monetary assets and that there is lower tail dependence between the demand for credit card transaction services and transaction balances.

JEL classification: C22; F33.
Keywords: Credit card-augmented Divisia monetary aggregates; Minflex Laurent functional form; Copulae.

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1 Introduction

Background. According to the 2019 Diary of Consumer Payment Choice, cash, credit cards, and debit cards have consistently made up approximately 95% of the overall payment preferences, with the growth rate of the preference for credit cards surpassing those for cash and other methods of payment. Indeed, the credit card is a leading payment method among all noncash payment methods in terms of both the number of transactions and total payment value. As shown in Figure 1, the use of credit card transaction services, increasing at 8% per year, has led the growth in the number of payments among noncash payment methods, and constituted more than 40% of the total number of the noncash transactions by 2018 (see the Federal Reserve Payments Study: 2019 Annual Supplement). Figure 2 shows that the total payment value of credit card transactions has grown steadily since the financial crisis of 2007-2009, with an annual growth rate of 10%.

Despite the widespread use of credit cards, credit card transaction services have only recently been included in monetary aggregates, because of accounting conventions, which do not permit adding liabilities, such as credit card balances, to assets, such as currency and demand deposits. However, Barnett *et al.* (2016) use economic aggregation theory and statistical index number theory to explicitly measure the service flows from credit card transactions and money and produce a new definitions of the money supply, the credit card-augmented Divisia monetary aggregates, currently available at the Center for Financial Stability (CFS). In Figures 3 and 4, we highlight the differences between the simple sum monetary aggregate (Sum M1), the conventional Divisia monetary aggregate (Divisia M1), and the credit card-augmented Divisia monetary aggregate (Divisia M1A) at the M1 level of monetary aggregation. As can be seen from Figures 3 and 4, the differences between the credit card-augmented Divisia M1A aggregate and the Sum M1 and conventional Divisia M1 aggregates are more pronounced during the Covid-19 crisis.

By including the joint (liquidity and transactions) services of credit cards and monetary assets, the new credit card-augmented CFS Divisia monetary aggregates seem conceptually more relevant to macroeconomic research as measures of monetary services in the economy, and can shed some light on the Barnett critique — the measurement problems associated with the failure to find significant relations between money and key macroeconomic variables. In this regard, Liu *et al.* (2020) use cyclical correlation analysis and Granger causality tests and find that during, and in the aftermath of the 2007-2009 global financial crisis, the credit card-augmented Divisia measures of money are more informative when predicting real economic activity than the conventional (CFS) Divisia monetary aggregates. Also, Liu and Serletis (2020) conclude that the balance sheet targeting monetary policies after the global financial crisis should pay more attention on the broad credit card-augmented Divisia aggregates to address economic and financial stability.

Contribution. The main objective of this paper is to study the demand for credit card transaction services and the liquidity services of traditional monetary assets (transactions balances and checkable deposits) over a sample period that includes the 2007-2009 global financial crisis and the Covid-19 pandemic. Since the credit card-augmented monetary aggregates are relatively new, no attempt has been made to systematically empirically investigate the substitutability/complementarity relationship between credit card transaction services and monetary assets. In this regard, Duca and Whitesell (1995) provide cross-sectional evidence that credit card ownership is associated with lower holdings of monetary transactions balances and Barnett *et al.* (2016, p. 2) conjecture that "credit card services are a substitute for the services of monetary transactions balances, perhaps to a much higher degree than the services of many of the assets included in traditional monetary aggregates."

We employ both 'nonparametric' and 'parametric' approaches to demand analysis. The nonparametric approach, fully developed by Varian (1982, 1983), deals with the raw data itself using techniques of finite mathematics. The parametric approach follows the innovative works by Diewert (1974), Donovan (1978), and Barnett (1978, 1980, 1983) and utilizes the flexible functional forms approach to investigating the inter-related problems of estimation of monetary asset demand functions and monetary aggregation. Our approach allows the estimation of a monetary asset demand system, augmented with credit card transactions services, based on the effectively globally regular Minflex Laurent flexible functional form, introduced by Barnett (1983) and Barnett and Lee (1985). We treat the concavity property as a maintained hypothesis to produce inference consistent with theoretical regularity.

A feature of all of the existing monetary asset demand studies is that they assume joint multivariate normality of the errors in the estimation of the demand system. See, for example, Ewis and Fisher (1984), Serletis and Robb (1986), Serletis (1988, 1991), Fisher and Fleissig (1997), Fleissig and Swofford (1997), Serletis and Shahmoradi (2005, 2007), and Jadidzadeh and Serletis (2019), among others. In this paper, we relax the joint normality assumption in the estimation of the Minflex Laurent flexible demand system by using the vine copula approach. The vine copula approach does not require a strictly-defined covariance matrix, and it allows the demand system errors to be from different families of distributions. By doing so, we are able to capture the various nonlinear dependence structures as well as tail dependence between the credit card transaction services and the monetary assets. Copulae have been widely used in the financial literature — see, for example, Patton (2006), Jondeau and Rockinger (2006), Rodriguez (2007), and Ning (2010) — and have been first introduced to the demand systems literature by Velasquez-Giraldo *et al.* (2018) and more recently by Serletis and Xu (2020b).

We show that the copula-based Minflex Laurent demand system provides a better fit to the data than when the model is estimated under the joint multivariate normality assumption. We find that credit card transaction services are substitutes for traditional monetary assets and there is lower tail dependence between the demand for credit card transaction services and transaction balances, as well as lower tail dependence between the demand for transaction balances and OCDs, meaning that during bad times, the dependencies between those pairs of monetary services are stronger. We find that most of the elasticities of substitution exhibit large swings during the global financial crisis of 2007-2009 and the Covid-19 pandemic. We also find the Minflex Laurent model, when estimated under the joint multivariate normality assumption, tends to underestimate the income elasticity of credit card transaction services, as well as the Morishima elasticities of substitution between credit card transaction services and monetary assets.

From the perspective of monetary policy, the substitutability between credit card transaction services and monetary assets we found in this paper provides evidence of the necessity to use the Divisia method of aggregation to include credit card transaction services into monetary aggregates. The current simple sum approach to monetary aggregation used by the Federal Reserve cannot include credit card transaction services into monetary aggregates due to accounting conventions. Moreover, the simple sum approach requires that the monetary aggregate components are perfect substitutes for each other and the elasticities of substitution between each other to be very high (perhaps infinite). The moderate elasticities of substitution we find between credit card transaction services and monetary assets provide empirical evidence of the superiority of the Divisia monetary aggregation method over the simple sum monetary aggregation method. We also find that the Morishima elasticity of substitution between transactions balances and credit card transaction services has remained relatively stable during the Covid-19 pandemic, but that most of the other Morishima elasticities of substitution declined during the pandemic, indicating that the asset demand functions have become more stable and predictable, enhancing the Fed's ability to target key monetary aggregates

Layout. The structure of the paper is as follows. Section 2 discusses the microeconomic foundations of the traditional Divisia monetary aggregates and the new credit card-augmented Divisia monetary aggregates. Section 3 presents the neoclassical monetary problem and Section 4 discusses the data. In Section 5, we use the nonparametric techniques of revealed preference analysis to test for consistency with preference maximization and the existence of consistent new credit card-augmented Divisia monetary aggregates. Section 6 presents the Minflex Laurent flexible functional form and the demand system. Section 7 discusses estimation issues and motivates the use of the copula method in the estimation of demand systems. In Section 8, we present the estimation results. Section 9 discusses the income and price elasticities and addresses the substitutability relationship between credit card transaction services and traditional monetary assets. Section 10 investigates the dynamics of the demand monetary services during the Covid-19 pandemic in terms of the time-varying Morishima elasticities of substitution. The final section concludes the paper.

2 Monetary Aggregation Issues

2.1 Simple-Sum Aggregates

Central banks around the world use the simple-sum index to construct monetary aggregates, as follows

$$M = \sum_{i=1}^{n} m_i^a$$

where M is the monetary aggregate and m_i^a is one of the n monetary assets of the monetary aggregate, M. Currently, the Federal Reserve constructs two monetary aggregates: the narrow Sum M1 and broad Sum M2 aggregates.

Although the simple-sum index has considerable advantages as an accounting measure of the stock of nominal monetary wealth, it has severe problems as a monetary aggregate index to track the liquidity services in the economy. Simple-summation monetary aggregation implies that all monetary components are perfect substitutes, and thus are equally weighted in the final liquidity measure. Simple-summation monetary aggregation cannot distinguish the changes in monetary service flow and the pure substitution between monetary components. In this regard, Fisher (1922) found the simple-sum index to be the worst known index number formula. The index number formula that Fisher found to be the best is the Fisher ideal index. Another attractive alternative to the simple-sum index is the (Tornqvist) discrete time approximation to the continuous Divisia index.

2.2 Divisia Aggregates

Over the years, Barnett (1978, 1980, 2016) argued that the simple-sum monetary aggregates are consistent with economic aggregation theory only if the monetary assets are perfect substitutes with the same user cost. However, monetary assets yield interest while currency does not. Thus, the assumption that the simple-sum monetary aggregates are based on is unreasonable. The Divisia monetary aggregates do not assume the perfect substitution between component assets, and hence permit different user costs of the component assets.

Because monetary assets are durable goods that do not perish during the period from use, their prices are their user costs. The formula for the real user cost of a monetary asset i, derived by Barnett (1978), can be written as

$$p_{it}^a = \frac{R_t - r_{it}^a}{1 + R_t} \tag{1}$$

where R_t is the benchmark asset rate of return, measuring the maximum expected rate of return available in the economy, and r_{it}^a is the own rate of return on monetary asset i during period t. The user cost can also be interpreted as the opportunity cost of holding a dollar's worth of the ith asset.

With the user cost and quantity data, the expenditure share on monetary asset i is

$$s_{it}^a = \frac{p_{it}^a m_{it}^a}{\sum_{i=1}^I p_{it}^a m_{it}^a} \tag{2}$$

where m_{it}^a denotes the real balances of monetary asset i during period t. A Divisia monetary aggregate computes the growth rate of the aggregate as the share-weighted average of its monetary asset component growth rates as follows

$$d \log M_t = \sum_{i=1}^I s_{it}^a d \log m_{it}^a \ . \tag{3}$$

Barnett (1978, 1980) demonstrated that the Divisia monetary aggregates represent a superior measurement of liquidity services compared to the simple-sum monetary aggregates. As a result, all the modern formal investigations of the impact of money on economic activities are carried out using the Divisia monetary aggregates, such as Belongia (1996), Serletis and Gogas (2014), Hendrickson (2013), Keating *et al.* (2019), Dai and Serletis (2019), Liu *et al.* (2020), and Dery and Serletis (2021), among others. All these works show that the Divisia monetary aggregates are superior to the simple-sum monetary aggregates in tracking liquidity services and have stronger explanatory power to economic activities. Moreover, Jadidzadeh and Serletis (2019) provide evidence that supports and reinforces Barnett's (2016) assertion that we should use, as a measure of money, the broadest Divisia M4 monetary aggregate, as opposed to narrower aggregates such as Divisia M1 or Divisia M2. All these studies emphasize that the choice of proper monetary measure matters in inference.

2.3 Credit Card-Augmented Divisia Aggregates

The volume of credit card transaction services has more than doubled in the past decade. Over 80% of American households with credit cards are currently borrowing and paying interest on credit cards (Barnett and Su (2019)). The simple-sum monetary aggregates are not able to include credit card transaction services due to accounting conventions. However, the Divisia monetary aggregates measure flows of services and are not based on accounting conventions. Using index number theory, the transaction services of credit cards and monetary assets can be aggregated jointly.

Barnett *et al.* (2016) derive the credit card-augmented Divisia monetary aggregates. Under the assumption of risk neutrality, they derive the user cost of credit card transaction services, p_{lt}^c, as

$$p_{lt}^c = \frac{e_{lt} - R_t}{1 + R_t} \tag{4}$$

where e_{lt} is the expected interest on the credit card transaction l and R_t is, as before, the rate of return on the benchmark asset. The credit card-augmented Divisia monetary aggregate is then computed (in growth rate form) as

$$d \log M_t = \sum_{i=1}^{I} s_{it}^{a} d \log m_{it}^{a} + \sum_{l=1}^{L} s_{lt}^{c} d \log m_{lt}^{c} \tag{5}$$

where

$$s_{it}^{a} = \frac{p_{it}^{a} m_{it}^{a}}{\sum_{i=1}^{I} p_{it}^{a} m_{it}^{a} + \sum_{l=1}^{L} p_{lt}^{c} m_{lt}^{c}}$$

is the user-cost-evaluated expenditure share of monetary asset, m_{it}^{a}, and

$$s_{lt}^{c} = \frac{p_{lt}^{c} m_{lt}^{c}}{\sum_{i=1}^{I} p_{it}^{a} m_{it}^{a} + \sum_{l=1}^{L} p_{lt}^{c} m_{lt}^{c}}$$

is the user-cost-evaluated expenditure share of credit card transaction services, m_{lt}^{c}.

The interest rate and risk on credit cards transactions are high and volatile. Barnett and Su (2019) and Barnett and Liu (2019) extend the theoretical credit card-augmented Divisia monetary aggregates under uncertainty, and more recently Barnett *et al.* (2019) further extend the existing theory of monetary services aggregation under risk to decisions under Knightian uncertainty. The credit card user cost under risk with intertemporal nonseparability is still ongoing research. The credit card augmented Divisia monetary aggregates supplied by the CFS program Advances in Monetary and Financial Measurement (AMFM) are based on the assumption of risk neutrality as derived by Barnett *et al.* (2016).

3 The Neoclassical Monetary Problem

We assume that the representative agent's utility function is

$$U = u\left(\boldsymbol{c},\, \ell,\, \boldsymbol{x}\right) \tag{6}$$

where $\boldsymbol{c}$ is a vector of the services of consumption goods, ℓ is leisure time, and $\boldsymbol{x}$ is a vector of the services of conventional monetary assets and credit cards, included in the broadest CFS credit card-augmented Divisia monetary aggregate, Divisia M4A, and described in Table 1. We assume that the agent faces the following maximization problem

$$\max_{\{\boldsymbol{c},\, \ell\,,\, \boldsymbol{x}\}} u\left(\boldsymbol{c},\, \ell,\, \boldsymbol{x}\right) \quad \text{subject to} \quad \boldsymbol{q}'\boldsymbol{c} + w\ell + \boldsymbol{p}'\boldsymbol{x} = I$$

where $\boldsymbol{q}'$ is a vector of the prices of the consumption goods, w is the price of leisure time (assumed to be the wage rate), and I is the expenditure on the services of consumption goods, leisure, and monetary services.

4 The Data

Two sets of data are used in our analysis. We use the total personal consumption expenditures (PCE) series and the corresponding (chain-type) price index (PCEPI) from the Federal Reserve Bank of St. Louis FRED data base. We also use the total private average weekly hours of production and nonsupervisory employees (AWHNONAG) series and the corresponding average hourly earnings (AHETPI) series from FRED. Regarding the data on monetary asset and credit card services, and their user costs, we use the recently produced data for the United States by Barnett *et al.* (2016), and maintained within the CFS program Advances in Monetary and Financial Measurement (AMFM), as shown in Table 1. For a detailed discussion of the CFS data and the methodology for the calculation of user costs, see Barnett *et al.* (2016) and http://www.centerforfinancialstability.org.

Since currency, traveler's checks, and demand deposits have the same user cost, we use simple summation to get the transactions balances subaggregate, x_1. Because OCDs at commercial banks and OCDs at thrift institutions have similar user costs, we use simple summation to get the OCDs subaggregate, x_2, and average the user costs of OCDs at commercial banks and OCDs at thrift institutions to get the user cost of the x_2 subaggregate, p_2. Constructing these subaggregates from the original monetary components enables us to reduce the dimension of the system. Since the credit card transaction services data, x_3, is available since 2006:7, our sample period is from 2006:7 to 2020:8 (a total of 170 monthly observations), and includes the extreme economic events of the 2007-2009 financial crisis and the 2020 Covid-19 crisis. Finally, as we require real per capita quantities for the empirical work, we divide each quantity series by the CPI (all items) and total population, both series obtained from the FRED data base. We also multiply the real user costs by the CPI to get nominal user costs.

5 Revealed Preference Analysis

In this section, we deal with the utility relation expressed in the direct form (6), and use the nonparametric approach to demand analysis, developed by Varian (1982, 1983). This approach deals with the raw data itself, consisting of observed prices and quantities for a set of consumer goods, using techniques of finite mathematics. We consider monthly data on leisure, ℓ, and real per capita data on consumption, c, and the 13 monetary assets and credit card transaction services shown in Table 1. We address three issues concerning consumer behavior: (i) consistency with the generalized axiom of revealed preference (GARP); (ii) consistency with the homothetic axiom of revealed preference (HARP); and (iii) weak separability of the representative agent's utility function. In doing so, we use the Demetry *et al.* (2020) *Stata* command `checkax` which implements Varian's (1982) algorithm. The command provides information regarding the number of observations that violate the hypothesis.

5.1 GARP Tests

The first problem considered is whether the utility maximization hypothesis could be established for consumption, leisure, and the CFS credit card-augmented definitions of money shown in Table 1 — M1A, M2MA, MZMA, M2A, ALLA, M3A, M4A-, and M4A. We use the Demetry *et al.* (2020) *Stata* command `checkax` to implement Varian's (1982) GARP test; see also Hjertstrand *et al.* (2016). The command provides information regarding the number of observations that violate the hypothesis. A data set $(p_t; x_t)_{t=1,...,T}$ satisfies GARP, if $x_t R_e x_s$ implies $p_s x_s \leq p_s x_t$. In a data set of T observations, the total possible number of GARP violations is $T(T-1)$ — see Demetry *et al.* (2020).

The results are presented in panel A of Table 2. We find no violations of the generalized axiom of revealed preference with the components of the MZMA, M2A, MALL, and M3A aggregates. The M4A- monetary aggregate has only one violation of the GARP test, the M1A and M2MA aggregates have three violations, while the M4A aggregate has nine violations.

5.2 HARP Tests

Because many money demand studies (based on the demand systems approach) have utilized homothetic functional forms for the underlying aggregator functions, we also test the homothetic axiom of revealed preference. Again, we use the Demetry *et al.* (2020) *Stata* command `checkax` to implement the HARP test as described in Varian (1983). In a data set of T observations, the total possible number of HARP violations is T.

Panel A of Table 2 shows that there are 170 violations for all the HARP tests for consumption, leisure, and monetary aggregates. Panel B shows that there are 170 violations for different levels of monetary aggregates as well. Clearly, the data is not consistent with homothetic preferences, a result consistent with the findings by Swofford and Whitney (1987, 1988) in quarterly and annual data for the United States and Serletis and Rangel-Ruiz (2005) in Canadian data.

5.3 Weak Separability Tests

Finally, we test a number of hypotheses to see if weakly separable groupings could be established for the CFS credit card-augmented definitions of money shown in Table 1 — M1A, M2MA, MZMA, M2A, ALLA, M3A, M4A-, and M4A. These results are presented in panel B of Table 2 (the number of GARP violations is reported in the table). Among all the monetary aggregates, the M1A aggregate has the smallest number of violations, that is five. All the other groupings of assets fail the separability tests with a larger number of violations, suggesting that the M2MA, MZMA, M2A, ALLA, M3A, M4A-, and M4A monetary aggregates are not weakly separable from consumption and leisure.

Considering that the nonparametric test used here to evaluate weak separability is biased toward rejecting separability and that the M1A monetary aggregate appears generally consistent with a representative economic agent maximizing a separable utility function, we assume that the economic agent's direct utility function (6) is weakly separable as follows

$$U = u\left(\boldsymbol{c}, \ell\ , g\left(x_1, x_2, x_3\right)\right)$$

so that we can focus on the details of the demand for the services of monetary assets and credit cards, ignoring the services of consumption goods, $\boldsymbol{c}$, and leisure, ℓ, as in the following monetary problem

$$\max_{\{x_1, x_2, x_3\}} g\left(x_1, x_2, x_3\right) \quad \text{subject to} \quad p_1 x_1 + p_2 x_2 + p_3 x_3 = y$$

where p_1, p_2, and p_3 are the user costs corresponding to x_1, x_2, and x_3, respectively, and y is the expenditure on the services of monetary assets and credit cards, x_1, x_2, and x_3, determined in the first stage (that of budgeting) of the (implicit) two-stage optimization.

In what follows, we use the parametric approach to demand analysis and investigate the substitutability/complementarity relation between traditional monetary assets and credit card transaction services. In particular, we model the demand system for the monetary assets that are included in the narrowest credit card-augmented M1A monetary aggregate — transaction balances (currency, traveler's checks, and demand deposits), x_1, other checkable deposits (OCDs) at commercial banks and thrift institutions, x_2, and credit card transaction services, x_3.

6 Parametric Demand Analysis

The parametric approach to demand analysis requires that we postulate parametric forms for the aggregator function and fit the derived demand functions to observed data. The estimated demand functions can then be used to estimate price and substitution elasticities. We use the indirect utility function to derive the demand system in budget share form, using Roy's identity, because our estimation is significantly simplified (see Barnett (1983)). Also, as Varian (1982, p. 945) put it, the parametric approach "will be satisfactory only when the postulated parametric forms are good approximations to the 'true' demand functions." We tackle this problem by using a flexible functional form.

6.1 The Minflex Laurent Flexible Functional Form

We follow Barnett (1983) and Barnett and Lee (1985) and use the Minflex Laurent reciprocal indirect utility function to approximate the dual direct utility function, $g\left(x_1, x_2, x_3\right)$.

The Minflex Laurent reciprocal indirect utility function is written as a function of income normalized prices, $\nu_i = p_i/y$, for $i = 1, 2, 3$, as

$$h(\boldsymbol{v}) = c + 2\boldsymbol{\delta}'\sqrt{\boldsymbol{v}} + \sum_{i=1}^{3} d_{ii}\nu_i + \sum_{i=1}^{3}\sum_{j=1, j\neq i}^{3} d_{ij}^2 \nu_i^{\frac{1}{2}}\nu_j^{\frac{1}{2}} - \sum_{i=1}^{3}\sum_{j=1, j\neq i}^{3} \beta_{ij}^2 \nu_i^{-\frac{1}{2}}\nu_j^{-\frac{1}{2}} \tag{7}$$

where $\boldsymbol{v}$ is the vector of income normalized prices, c is a constant, and $\boldsymbol{\delta} = (\delta_1, \delta_2, \delta_3)'$, d_{ij} and β_{ij} are all parameters.

By applying Roy's identity, the share equations of the monetary assets and credit card transaction services can be obtained

$$s_i = \frac{\delta_i \nu_i^{\frac{1}{2}} + d_{ii}\nu_i + \sum\limits_{j=1, j\neq i}^{n} d_{ij}^2 \nu_i^{\frac{1}{2}}\nu_j^{\frac{1}{2}} + \sum\limits_{j=1, j\neq i}^{n} \beta_{ij}^2 \nu_i^{-\frac{1}{2}}\nu_j^{-\frac{1}{2}}}{\boldsymbol{\delta}'\sqrt{\boldsymbol{v}} + \sum\limits_{i=1}^{n} d_{ii}\nu_i + \sum\limits_{i=1}^{n}\sum\limits_{j=1, j\neq i}^{n} d_{ij}^2 \nu_i^{\frac{1}{2}}\nu_j^{\frac{1}{2}} + \sum\limits_{i=1}^{n}\sum\limits_{j=1, j\neq i}^{n} \beta_{ij}^2 \nu_i^{-\frac{1}{2}}\nu_j^{-\frac{1}{2}}} \tag{8}$$

for $i = 1, 2, 3$. Because the share equations are homogeneous of degree zero in the parameters, a normalizing restriction is needed (see Barnett (1983)). We follow Barnett and Lee (1985) and impose the normalization

$$\sum_{i=1}^{3} d_{ii} + 2\sum_{i=1}^{3}\delta_i + \sum_{i=1}^{3}\sum_{j=1, j\neq i}^{3} d_{ij}^2 - \sum_{i=1}^{3}\sum_{j=1, j\neq i}^{3} \beta_{ij}^2 = 1.$$

Barnett (1983) has shown that the above reciprocal indirect utility function has more free parameters than is needed to acquire local flexibility in the Diewert (1976) sense. To reduce the number of free parameters, without losing the flexibility property, we follow Barnett (1983) and impose the following restrictions

$$d_{ij} = d_{ji}\ , \quad \beta_{ij} = \beta_{ji}, \quad d_{ij}\beta_{ij} = 0, \quad i \neq j.$$

Therefore, we obtain the minimal of the Minflex Laurent model, in the sense that imposing any further restrictions would eliminate its flexibility property.

6.2 Elasticity Measures

In the demand systems approach to the estimation of economic relationships, the primary interest, especially in policy analysis, is in the elasticity measures. Once the demand system is estimated, we can calculate different elasticity measures from the Marshallian demand functions, $x_i(\boldsymbol{v})$, $i = 1, 2, 3$ — see Barnett and Serletis (2008) for more details. In particular, to assess how changes in expenditure affect the quantities demanded for each asset, for each asset we compute the income elasticity, η_{iy}, as

$$\eta_{iy} = \frac{y}{x_i}\frac{\partial x_i}{\partial y}.$$

We can also compute the Marshallian demand elasticities, η_{ij}, as

$$\eta_{ij} = \frac{\partial x_i}{\partial v_j}\frac{v_j}{x_i}, \quad i, j = 1, 2, 3.$$

In addition, we can use the Allen-Uzawa and Morishima elasticities of substitution to investigate substitutability/complementarity relationships among the assets. The Allen elasticity provides comparative-static information about the effect of price changes on absolute demand shares. Following Serletis and Feng (2010), the Allen partial elasticity of substitution can be calculated as

$$\sigma_{ij}^{a} = \eta_{iy} + \frac{\eta_{ij}}{s_j}$$

where s_j is the share of asset x_j. If $\sigma_{ij}^{a} > 0$ (that is, an increase in the price of x_j induces an increase in the optimal quantity demanded of x_i), then x_i and x_j are Allen-Uzawa (net) substitutes. Alternatively, if $\sigma_{ij}^{a} < 0$, then they are Allen-Uzawa (net) complements. The Allen elasticities are symmetric, in other words, $\sigma_{ij}^{a} = \sigma_{ji}^{a}$, for all i and j.

However, the Allen-Uzawa elasticity of substitution may be uninformative in the case with more than two goods — see Blackorby and Russell (1989) for more details. In that case the Morishima elasticity of substitution is the correct measure of substitution. The Morishima elasticity of substitution, σ_{ij}^{m}, can be calculated as

$$\sigma_{ij}^{m} = s_j(\sigma_{ij}^{a} - \sigma_{jj}^{a})$$

and looks at the impact on the ratio of two assets, x_i/x_j, when there is a change in the price of asset j. Assets will be Morishima substitutes ($\sigma_{ij}^{m} > 0$) if an increase in p_j causes x_i/x_j to increase and Morishima complements ($\sigma_{ij}^{m} < 0$) if an increase in p_j causes x_i/x_j to decrease.

7 Estimation Matters

7.1 Stochastic Specification

In order to estimate the demand system (8), a stochastic version is specified, assuming that the observed share in the ith equation deviates from the true share by a disturbance term

ε_i. It is typically assumed that $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t})'$ is a vector of disturbance terms from a jointly normal distribution. Then the density function of $\boldsymbol{s} = (s_1, s_2, s_3)$ is

$$f(\boldsymbol{s}) = \prod_{i=1}^{3} f\left(s_i - \frac{\delta_i \nu_i^{\frac{1}{2}} + d_{ii}\nu_i + \sum\limits_{j=1, j\neq i}^{k} d_{ij}^2 \nu_i^{\frac{1}{2}} \nu_j^{\frac{1}{2}} + \sum\limits_{j=1, j\neq i}^{k} \beta_{ij}^2 \nu_i^{-\frac{1}{2}} \nu_j^{-\frac{1}{2}}}{\boldsymbol{\delta}' \sqrt{\boldsymbol{v}} + \sum\limits_{i=1}^{k} d_{ii}\nu_i + \sum\limits_{i=1}^{k} \sum\limits_{j=1, j\neq i}^{k} d_{ij}^2 \nu_i^{\frac{1}{2}} \nu_j^{\frac{1}{2}} + \sum\limits_{i=1}^{k} \sum\limits_{j=1, j\neq i}^{k} \beta_{ij}^2 \nu_i^{-\frac{1}{2}} \nu_j^{-\frac{1}{2}}} \right)$$

$$= f(\varepsilon_1) f(\varepsilon_2) f(\varepsilon_3) \tag{9}$$

where $f(\varepsilon_i)$ is the density function of ε_i. The corresponding log-likelihood function is

$$\mathcal{L}(\boldsymbol{\theta}, f) = \ln f(\varepsilon_1) + \ln f(\varepsilon_2) + \ln f(\varepsilon_3) \tag{10}$$

where $\boldsymbol{\theta}$ represents all the parameters in the Minflex Laurent demand system.

7.2 Traditional Estimation

The traditional approach to demand systems estimation further assumes that the elements of $\boldsymbol{\varepsilon}_t$ follow a joint standard normal distribution. Because the sum of the shares equals 1, the demand system as shown in equation (8) is a singular system and there is singularity in the covariance matrix of the residuals under the joint normality assumption. The singularity of the distribution of $\boldsymbol{\varepsilon}_t$ is due to the fact that the components of $\boldsymbol{\varepsilon}_t$ identically add up to zero. It is also to be noted that recently Serletis and Xu (2020a) address the estimation of singular demand systems with heteroscedastic disturbances, relaxing the homoscedasticity assumption and instead assuming that the covariance matrix of the errors of the demand system is time-varying.

Since Barten (1969), to estimate the demand system and the corresponding log-likelihood function (10), one of the share equations in (8) is arbitrarily deleted. That is, one of $\ln f(\varepsilon_i)$ is deleted from the log-likelihood function (10) due to the assumption of joint normality and the resulting vector has a non-singular distribution. As Barten (1969) shows, under the joint normality assumption, the parameter estimates obtained by trace minimization are invariant to the equation deleted.

7.3 A Copula Approach

The joint normality assumption is restrictive. For monetary assets and credit card transaction services, it is likely that there are dependencies among the disturbance terms, ε_i, $i = 1, 2, 3$, and, moreover, such dependence structures could be nonlinear. Copulae are a powerful tool for modelling nonlinear dependence between random variables, and in particular dependence in the tails of the distributions (known as 'tail dependence').

According to Sklar (1973), copulae can be used to express a multivariate distribution in terms of its marginal distributions. In particular, we can use copulae to piece together joint distributions when only marginal distributions are known (Trivedi and Zimmer (2007 p. 11)). Let $F_{12}(\varepsilon_1, \varepsilon_2)$ be an unknown joint distribution function of $(\varepsilon_1, \varepsilon_2)$. Then there is a unique copula function, C, such that

$$F_{12}(\varepsilon_1, \varepsilon_2) = C\left(F_1(\varepsilon_1), F_2(\varepsilon_2)\right) = C(u_1, u_2)$$

where $u_1 = F_1(\varepsilon_1)$ and $u_2 = F_2(\varepsilon_2)$ are the marginal cumulative distribution functions of ε_1 and ε_2, respectively. The joint density function $f_{12}(\varepsilon_1, \varepsilon_2)$ is

$$\begin{aligned} f_{12}(\varepsilon_1, \varepsilon_2) &= \frac{\partial^2 F_{12}(\varepsilon_1, \varepsilon_2)}{\partial \varepsilon_1 \partial \varepsilon_2} \\ &= \frac{\partial^2 C(u_1, u_2)}{\partial u_1 \partial u_2} \frac{\partial F_1(\varepsilon_1)}{\partial \varepsilon_1} \frac{\partial F_2(\varepsilon_2)}{\partial \varepsilon_2} \\ &= c(u_1, u_2) f_1(\varepsilon_1) f_2(\varepsilon_2) \end{aligned} \tag{11}$$

where $c(u_1, u_2) = \partial^2 C(u_1, u_2)/\partial u_1 \partial u_2$ and $f_1(\varepsilon_1)$ and $f_2(\varepsilon_2)$ are the probability density functions of $F_1(\varepsilon_1)$ and $F_2(\varepsilon_2)$, respectively. For independent copulae, $C(u_1, u_2) = u_1 u_2$ and $c(u_1, u_2) = 1$.

Let $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2, \varepsilon_3) \sim F$ with marginal distribution functions $F_1(\varepsilon_1)$, $F_2(\varepsilon_2)$, and $F_3(\varepsilon_3)$ and the corresponding density functions $f_1(\varepsilon_1)$, $f_2(\varepsilon_2)$, and $f_3(\varepsilon_3)$, respectively. According to Aas and Berg (2009, p. 6) and Brechmann and Schepsmeier (2013, p. 3), by recursive conditioning we can write

$$f(\varepsilon_1, \varepsilon_2, \varepsilon_3) = f_3(\varepsilon_3) f(\varepsilon_2|\varepsilon_3) f(\varepsilon_1|\varepsilon_2, \varepsilon_3) \tag{12}$$

$$f(\varepsilon_2|\varepsilon_3) = \frac{f(\varepsilon_2, \varepsilon_3)}{f_3(\varepsilon_3)} \tag{13}$$

$$f(\varepsilon_1|\varepsilon_2, \varepsilon_3) = \frac{f(\varepsilon_1, \varepsilon_2, \varepsilon_3)}{f_3(\varepsilon_3) f(\varepsilon_2|\varepsilon_3)} = \frac{f(\varepsilon_1, \varepsilon_2|\varepsilon_3)}{f(\varepsilon_2|\varepsilon_3)}. \tag{14}$$

By Sklar's theorem, (11), we have

$$f(\varepsilon_2, \varepsilon_3) = c_{2,3}(F_2(\varepsilon_2), F_3(\varepsilon_3)) f_2(\varepsilon_2) f_3(\varepsilon_3). \tag{15}$$

Plugging equation (15) into equation (13) yields

$$\begin{aligned} f(\varepsilon_2|\varepsilon_3) &= \frac{f(\varepsilon_2, \varepsilon_3)}{f_3(\varepsilon_3)} \\ &= \frac{c_{2,3}(F_2(\varepsilon_2), F_3(\varepsilon_3)) f_2(\varepsilon_2) f_3(\varepsilon_3)}{f_3(\varepsilon_3)} \\ &= c_{2,3}(F_2(\varepsilon_2), F_3(\varepsilon_3)) f_2(\varepsilon_2). \end{aligned} \tag{16}$$

Similarly, from equation (14) and equation (11), we obtain

$$
\begin{aligned}
f(\varepsilon_1|\varepsilon_2,\varepsilon_3) &= \frac{f\left(\varepsilon_1,\varepsilon_2|\varepsilon_3\right)}{f(\varepsilon_2|\varepsilon_3)} \\
&= \frac{c_{1,2|3}(F(\varepsilon_1|\varepsilon_3),F(\varepsilon_2|\varepsilon_3))f(\varepsilon_1|\varepsilon_3)f(\varepsilon_2|\varepsilon_3)}{f(\varepsilon_2|\varepsilon_3)} \\
&= c_{1,2|3}(F(\varepsilon_1|\varepsilon_3),F(\varepsilon_2|\varepsilon_3))f(\varepsilon_1|\varepsilon_3) \\
&= c_{1,2|3}(F(\varepsilon_1|\varepsilon_3),F(\varepsilon_2|\varepsilon_3))\frac{f(\varepsilon_1,\varepsilon_3)}{f_3(\varepsilon_3)} \\
&= c_{1,2|3}(F(\varepsilon_1|\varepsilon_3),F(\varepsilon_2|\varepsilon_3))\frac{c_{1,3}(F_1(\varepsilon_1),F_3(\varepsilon_3))f_1(\varepsilon_1)f_3(\varepsilon_3)}{f_3(\varepsilon_3)} \\
&= c_{1,2|3}(F(\varepsilon_1|\varepsilon_3),F(\varepsilon_2|\varepsilon_3))c_{1,3}(F_1(\varepsilon_1),F_3(\varepsilon_3))f_1(\varepsilon_1) \qquad (17)
\end{aligned}
$$

where $F(\varepsilon_1|\varepsilon_3) = \partial C_{13}(\varepsilon_1,\varepsilon_3)/\partial\varepsilon_3$ and $F(\varepsilon_2|\varepsilon_3) = \partial C_{23}(\varepsilon_2,\varepsilon_3)/\partial\varepsilon_3$ — see Aas and Berg (2009, p. 6). The three-dimensional joint density as shown in equation (12) can therefore be represented in terms of bivariate copulae $C_{1,3}$, $C_{2,3}$, and $C_{1,2|3}$ with densities $c_{1,3}$, $c_{2,3}$, and $c_{1,2|3}$, respectively. These pair-copulae can be chosen independently of each other to achieve a wide range of different dependence structures. By carefully choosing component copulae $C_{1,3}$, $C_{2,3}$, and $C_{1,2|3}$ and their mixture, we can construct a model that is simple yet flexible enough to generate most dependence patterns in the data.

The density function for the Minflex Laurent copula demand system can be derived by plugging equations (16) and (17) into equation (12) to obtain

$$
\begin{aligned}
f(\varepsilon_1,\varepsilon_2,\varepsilon_3) = {} & f_1(\varepsilon_1)f_2(\varepsilon_2)f_3(\varepsilon_3) \\
& \times c_{2,3}(F_2(\varepsilon_2),F_3(\varepsilon_3)) \\
& \times c_{1,3}(F_1(\varepsilon_1),F_3(\varepsilon_3))c_{1,2|3}(F(\varepsilon_1|\varepsilon_3),F(\varepsilon_2|\varepsilon_3)). \qquad (18)
\end{aligned}
$$

The corresponding log-likelihood function is

$$
\begin{aligned}
\mathcal{L}\left(\boldsymbol{\theta},f;\ \boldsymbol{\alpha}\right) = {} & \ln f_1(\varepsilon_1) + \ln f_2(\varepsilon_2) + \ln f_3(\varepsilon_3) \\
& + \ln c_{2,3}(F_2(\varepsilon_2),F_3(\varepsilon_3)) \\
& + \ln c_{1,3}(F_1(\varepsilon_1),F_3(\varepsilon_3)) + \ln c_{1,2|3}(F(\varepsilon_1|\varepsilon_3),F(\varepsilon_2|\varepsilon_3)) \qquad (19)
\end{aligned}
$$

where $\boldsymbol{\theta}$ represents (as before) all the parameters in the Minflex Laurent demand system and $\boldsymbol{\alpha}$ the parameters in the copulae functions. We assume that each of ε_1, ε_2, and ε_3 follows a univariate normal distribution, that is $\varepsilon_1 \sim N(0,\sigma_1^2)$, $\varepsilon_2 \sim N(0,\sigma_2^2)$, and $\varepsilon_3 \sim N(0,\sigma_3^2)$.

According to Sklar's theorem (see equation (11)), when ε_1, ε_2, and ε_3 are independent, $c_{1,3}(F_1(\varepsilon_1),F_3(\varepsilon_3))$, $c_{2,3}(F_2(\varepsilon_2),F_3(\varepsilon_3))$, and $c_{1,2|3}(F(\varepsilon_1|\varepsilon_3),F(\varepsilon_2|\varepsilon_3))$ all equal to 1. When each of ε_1, ε_2, and ε_3 is following a standard normal distribution and ε_1, ε_2, and ε_3 are independent of each other, the joint density function in equation (18) collapses to (9) and the log-likelihood function (19) collapses to (10). Therefore, the estimation of the Minflex Laurent demand system under the joint normality assumption for the errors is a special case of the Minflex Laurent copula demand system.

As Berndt and Savin (1975, p. 937) put it, "by singular equation systems we mean systems in which the sum of the regressands at each observation is equal to a linear combination of certain regressors." By relaxing the assumption of joint normality in the disturbance terms, we allow nonlinear dependence across the disturbance terms of the demand system equations, and the sum of the regressands at each observation is no longer a linear combination of certain regressors, but a nonlinear combination of certain regressors. Therefore, the distribution of $\boldsymbol{\varepsilon}_{\ t}$ is not singular when we allow nonlinear dependence across the disturbance terms of the demand system equations. Thus, we do not need to arbitrarily delete any equation in our maximum likelihood estimation of equations (8) to get a non-singular distribution.

The estimates of our Minflex Laurent copula demand system are obtained by solving the score equations $\partial\mathcal{L}/\partial\boldsymbol{\phi} = \mathbf{0}$, where $\boldsymbol{\phi} = (\boldsymbol{\theta}, \boldsymbol{\alpha})$ represents all the parameters in the demand system and the copulae. These equations will be nonlinear in general, but standard quasi-Newton iterative algorithms are available in most matrix programming languages. Let the solution be $\widehat{\boldsymbol{\phi}}_{FML}$. According to Trivedi and Zimmer (2007 p. 57), by standard likelihood theory under regularity conditions, $\widehat{\boldsymbol{\phi}}_{FML}$ is consistent for the true parameter vector $\boldsymbol{\phi}$ and its asymptotic distribution is given by

$$\sqrt{n}\left(\widehat{\boldsymbol{\phi}}_{FML} - \boldsymbol{\phi}\right) \xrightarrow{d} \mathcal{N}\left[\mathbf{0}, -\left(p\lim \frac{1}{n}\frac{\partial^2 L\left(\boldsymbol{\phi}\ \right)}{\partial\boldsymbol{\phi}\partial\boldsymbol{\phi}'}\right)_{\boldsymbol{\phi}}^{-1}\right]. \tag{20}$$

Quasi-likelihood estimation is preferred as it allows for possible misspecification of the copula and the "sandwich" variance estimator is consistent.

The construction of the three-dimensional dependence copula we described above from equations (12)-(18) is called pair copula construction (PCC), originally proposed by Joe (1996) and later discussed in detail by Bedford and Cooke (2001, 2002) and Kurowicka and Cooke (2006). The PCC is hierarchical in nature. The modelling scheme is based on a decomposition of a multivariate density into a cascade of bivariate copulae densities. For a κ-dimensional problem, the PCC allows for the free specification of $\kappa(\kappa - 1)/2$ copulae. The bivariate copulae may be from any family and several families may well be mixed in one PCC.

It is to be noted that a different method for building higher-dimensional copulae, the nested Archimedean construction (NAC), is also commonly used. For example, Serletis and Xu (2020b) use NAC in their investigation of interfuel substitution in the United States with the Minflex Laurent demand system. However, the NAC only allows for the modelling of up to $\kappa - 1$ bivariate copulae. Aas and Berg (2009) compare the nested Archimedean construction and the pair-copula construction and show that the former is much more restrictive than the latter in two respects. In particular, the NAC has strong limitations on the degree of dependence in each level of the NAC, and all the bivariate copulae in this construction have to be Archimedean. They claim that the PCC is more suitable than the NAC for high-dimensional modelling.

The ways to write a trivariate probability density function in terms of the conditional probability density functions and univariate probability density functions are not unique. We first have to choose which variables to join at the first level of the PCC. According to Aas and Berg (2009, p. 646), in empirical analysis, we usually join the variables that have the strongest tail dependence first. Having chosen the order of the variables at the first level, we then can determine which factorization to use. We discuss the steps of choosing copulae in detail in Section 8.2.

8 Empirical Evidence

8.1 ...Based on Traditional Estimation

All the estimations are performed in RATS 10.0. We first use the maximum log-likelihood estimation method to estimate the Minflex Laurent demand system based on the joint standard normality distribution assumption as shown in equation (10). We present the parameter estimates (together with p-values) in column 2 of Table 3. To check whether the assumption of joint normality of the residuals holds, we perform the Shapiro-Wilk (1965) normality test. The data reject the null hypothesis of the joint normal distribution of ε_1, ε_2, and ε_3 with a p-value of 0.000. In other words, ε_1, ε_2, and ε_3 are not jointly independent but are correlated.

In column 2 of Table 4 we present the linear correlation coefficients of the $(\varepsilon_1, \varepsilon_2)$, $(\varepsilon_1, \varepsilon_3)$, and $(\varepsilon_2, \varepsilon_3)$ pairs, conditional on the estimates of $\boldsymbol{\theta}$ in equation (10). As can be seen, the linear correlation coefficients of the $(\varepsilon_1, \varepsilon_2)$, $(\varepsilon_1, \varepsilon_3)$, and $(\varepsilon_2, \varepsilon_3)$ pairs are -0.757, 0.024, and -0.671, respectively. We find a sharp increase in the linear correlation coefficients of $(\varepsilon_1, \varepsilon_3)$ and $(\varepsilon_2, \varepsilon_3)$ over the recession periods compared to the correlation coefficients over the non-recession period. Moreover, the sign of the correlation between ε_1 and ε_3 switched from 0.087 in the non-recession period to -0.524 in the recession period.

Poon *et al.* (2004) summarizes that the conventional dependence measure, the linear correlation coefficient, calculated as the average of deviations from the mean, assumes a linear relationship between variables which follow a joint Gaussian distribution. The risk of joint extreme events could be underestimated. Moreover, it cannot distinguish between positive and negative returns, neither the large nor small values. Alternatively, both Kendall's τ and Spearman's ρ statistics can describe the nonlinear tail dependence structure. Kendall's τ is defined as

$$\rho_\tau(\varepsilon_1, \varepsilon_2) = \Pr[(\varepsilon_1^1 - \varepsilon_1^2)(\varepsilon_2^1 - \varepsilon_2^2) > 0] - \Pr[(\varepsilon_1^1 - \varepsilon_1^2)(\varepsilon_2^1 - \varepsilon_2^2) < 0]$$

where $(\varepsilon_1^1, \varepsilon_2^1)$ and $(\varepsilon_1^2, \varepsilon_2^2)$ are two independent pairs of random variables. The first term on the right, $\Pr[(\varepsilon_1^1 - \varepsilon_1^2)(\varepsilon_2^1 - \varepsilon_2^2) > 0]$, is referred to as Pr[concordance] and the second term, $\Pr[(\varepsilon_1^1 - \varepsilon_1^2)(\varepsilon_2^1 - \varepsilon_2^2) < 0]$, as Pr[discordance]. Thus,

$$\rho_\tau(\varepsilon_1, \varepsilon_2) = \Pr[\text{concordance}] - \Pr[\text{discordance}]$$

is a measure of the relative difference between the two random variables. Spearman's ρ is defined as

$$\rho_s(\varepsilon_1, \varepsilon_2) = \rho(F_1(\varepsilon_1), F_2(\varepsilon_2))$$

where ε_1 and ε_2 are two random variables, and F_1 and F_2 are the corresponding distribution functions. Spearman's ρ is the linear correlation between $F_1(\varepsilon_1)$ and $F_2(\varepsilon_2)$, which are integral transforms of ε_1 and ε_2. Both Kendall's τ and Spearman's ρ use the rankings of the data to measure the relationship between two variables, while the linear correlation coefficient uses actual values to measure the relationship between the two variables. As demonstrated by Embrechts *et al.* (2002), rank correlations are more robust measures of dependence than linear correlation.

In columns 3 and 4 of Table 4 we report the Kendall's τ and Spearman's ρ rank correlation coefficients, respectively. As can be seen, they are different across pairs. There is negative dependence for all the pairs and the dependence is the strongest between ε_1 and ε_2, with Kendall's τ of -0.602 and Spearman's ρ of -0.824. The potential source of dependence can be unmeasured factors such as shocks and innovations in the demand of each good. To explore the dependence structure and the choice of the appropriate copula to use, we scatter plot the $(\varepsilon_1, \varepsilon_2)$, $(\varepsilon_1, \varepsilon_3)$, and $(\varepsilon_2, \varepsilon_3)$ pairs in Figures 5-7. Clearly, the $(\varepsilon_1, \varepsilon_2)$ and $(\varepsilon_1, \varepsilon_3)$ pairs exhibit negative dependence. Moreover, Figures 5 and 6 show that the observations are clustered at the very left and these clusters are quite sizeable, indicating there could be lower tail dependence in the $(\varepsilon_1, \varepsilon_2)$ and $(\varepsilon_1, \varepsilon_3)$ pairs. Figure 7 shows slight right tail dependence; the clustering of observations and the tail dependence structure are less obvious than those of Figures 5 and 6.

8.2 ...Based on the Copula Approach

An appropriate copula to use is one which best captures dependence features of the outcome variables. The important consideration when selecting an appropriate copula is whether dependence is accurately represented. Since Figures 5 and 6 show clear lower tail dependence, copulae that can only capture upper tail dependence, such as the Gumbel (1960) and Joe (1993) copulae might be inappropriate. In what follows , we focus on the Clayton (1978) copula, the Frank (1979) copula, and the mixture of the Clayton and Frank copulae. As the Clayton copula is only able to capture positive dependence, to be able to capture the negative dependence of the $(\varepsilon_1, \varepsilon_2)$ and $(\varepsilon_1, \varepsilon_3)$ pairs, we transform $F_1(\varepsilon_1)$ to $1 - F_1(\varepsilon_1)$, where F_1 is the cdf of $\varepsilon_1 \sim N(0, \sigma_1^2)$.

It is also important to note at this stage that the dependence measures discussed so far are conditional on the explanatory variables and parameter estimates of the Minflex Laurent demand system from equations (8) and (10). Consequently, dependence conditional on the explanatory variables and parameter estimates of the Minflex Laurent copula demand system as shown in equation (19) can be different. As pointed out by Trivedi and Zimmer (2007 p. 79), "a valid empirical approach is to estimate several different copulae and choose the model that yields the largest penalized log-likelihood value." In what follows, we choose the AIC value as a measure of the goodness of fit.

8.2.1 Clayton Copula

Consider the pair $(\varepsilon_1, \varepsilon_2)$, where $\varepsilon_1 \sim N(0, \sigma_1^2)$, $\varepsilon_2 \sim N(0, \sigma_2^2)$, and $u_1 = F_1(\varepsilon_1)$ and $u_2 = F_2(\varepsilon_2)$ are the cumulative distribution functions of ε_1 and ε_2, respectively. The bivariate Clayton copula of $(\varepsilon_1, \varepsilon_2)$ is

$$C(u_1, u_2, \alpha) = (u_1^{-\alpha} + u_2^{-\alpha} - 1)^{-1/\alpha}. \tag{21}$$

The probability density function (pdf) for the bivariate Clayton copula is

$$c(u_1, u_2; \alpha) = \frac{(1+\alpha)(u_1^{-\alpha} + u_2^{-\alpha} - 1)^{-\frac{1}{\alpha}-2}}{(u_1 u_2)^{\alpha+1}} \tag{22}$$

where $\alpha \geq 0$. The Clayton copula can capture positive lower tail dependence but cannot capture negative dependence nor upper tail dependence.

Given the functional form of the Clayton copula, as shown in equations (21) and (22), using pair-copula construction the pdf of the trivariate Clayton copula can be derived according to equation (18). Specifically, according to equation (22), we have

$$c_{1,3}(F_1(\varepsilon_1), F_2(\varepsilon_3)) = \frac{(1+\alpha_1)(u_1^{-\alpha_1} + u_3^{-\alpha_1} - 1)^{-\frac{1}{\alpha_1}-2}}{(u_1 u_3)^{\alpha_1+1}} \tag{23}$$

and

$$c_{2,3}(F_1(\varepsilon_2), F_3(\varepsilon_3)) = \frac{(1+\alpha_2)(u_2^{-\alpha_2} + u_3^{-\alpha_2} - 1)^{-\frac{1}{\alpha_2}-2}}{(u_2 u_3)^{\alpha_2+1}} \tag{24}$$

where $u_i = F_i(\varepsilon_i)$ and F_i is the cumulative distribution function (cdf) of $\varepsilon_i \sim N(0, \sigma_i^2)$ for $i = 1, 2, 3$. α_1 and α_2 are the dependence parameters in $c_{1,3}$ and $c_{2,3}$. Based on equation (22), we also have

$$c_{1,2|3}(F(\varepsilon_1|\varepsilon_3), F(\varepsilon_2|\varepsilon_3)) = \frac{(1+\alpha_3)(u_4^{-\alpha_3} + u_5^{-\alpha_3} - 1)^{-\frac{1}{\alpha_3}-2}}{(u_4 u_5)^{\alpha_3+1}} \tag{25}$$

where

$$u_4 = F(\varepsilon_2|\varepsilon_3) = \frac{\partial C_{23}(\varepsilon_2, \varepsilon_3)}{\partial \varepsilon_3} = u_3^{-\alpha_2 - 1}(u_3^{-\alpha_2} + u_2^{-\alpha_2} - 1)^{-1/\alpha_2 - 1}$$

and

$$u_5 = F(\varepsilon_1|\varepsilon_3) = \frac{\partial C_{13}(\varepsilon_1, \varepsilon_3)}{\partial \varepsilon_3} = u_3^{-\alpha_1 - 1}(u_3^{-\alpha_1} + u_1^{-\alpha_1} - 1)^{-1/\alpha_1 - 1}.$$

Plugging equations (23), (24), and (25) into (18), yields the pdf function of the Minflex Laurent Clayton copula demand system

$$\begin{aligned} f(\varepsilon_1, \varepsilon_2, \varepsilon_3) &= f_1(\varepsilon_1) f_2(\varepsilon_2) f_3(\varepsilon_3) \times c_{1,3}(F_1(\varepsilon_1), F_3(\varepsilon_3)) \\ &\times c_{2,3}(F_2(\varepsilon_2), F_3(\varepsilon_3)) \times c_{1,2|3}(F(\varepsilon_1|\varepsilon_3), F(\varepsilon_2|\varepsilon_3)) \\ &= f_1(\varepsilon_1) f_2(\varepsilon_2) f_3(\varepsilon_3) \times \frac{(1+\alpha_1)(u_1^{-\alpha_1} + u_3^{-\alpha_1} - 1)^{-\frac{1}{\alpha_1} - 2}}{(u_1 u_3)^{\alpha_1 + 1}} \\ &\times \frac{(1+\alpha_2)(u_2^{-\alpha_2} + u_3^{-\alpha_2} - 1)^{-\frac{1}{\alpha_2} - 2}}{(u_2 u_3)^{\alpha_2 + 1}} \times \frac{(1+\alpha_3)(u_4^{-\alpha_3} + u_5^{-\alpha_3} - 1)^{-\frac{1}{\alpha_3} - 2}}{(u_4 u_5)^{\alpha_3 + 1}} \end{aligned} \tag{26}$$

where $\varepsilon_i \sim N(0, \sigma_i^2)$, $f_i(\varepsilon_i)$ are the pdfs, and $u_i = F_i(\varepsilon_i)$ are the corresponding cdfs of ε_i, $i = 1, 2, 3$. α_i are the copulae dependence parameters.

8.2.2 Frank Copula

The cumulative distribution function of the Frank (1979) copula of $(\varepsilon_1, \varepsilon_2)$ is

$$C(u_1, u_2; \alpha) = -\alpha^{-1} \ln \left(\frac{1 - e^{-\alpha} - (1 - e^{-\alpha u_1})(1 - e^{-\alpha u_2})}{1 - e^{-\alpha}} \right).$$

The pdf for the Frank copula is

$$c(u_1, u_2; \alpha) = \frac{\alpha(1 - e^{-\alpha}) e^{-\alpha(u_1 + u_2)}}{[1 - e^{-\alpha} - (1 - e^{-\alpha u_1})(1 - e^{-\alpha u_2})]^2}$$

where the dependence parameter α can capture symmetric positive lower and upper tail dependence. In particular, values of $\alpha < 0$ and $\alpha > 0$ correspond to negative and positive dependence, respectively. When α approaches 0, u_1 and u_2 are independent.

Following a similar procedure as for the Clayton copula, we can obtain the pdf function of the Minflex Laurent Frank copula demand system

$$
\begin{aligned}
f(\varepsilon_1, \varepsilon_2, \varepsilon_3) &= f_1(\varepsilon_1) f_2(\varepsilon_2) f_3(\varepsilon_3) \times c_{1,3}(F_1(\varepsilon_1), F_3(\varepsilon_3)) \\
&\quad \times c_{2,3}(F_2(\varepsilon_2), F_3(\varepsilon_3)) \times c_{1,2|3}(F(\varepsilon_1|\varepsilon_3), F(\varepsilon_2|\varepsilon_3)) \\
&= f_1(\varepsilon_1) f_2(\varepsilon_2) f_3(\varepsilon_3) \times \frac{\alpha_1(1-e^{-\alpha_1})e^{-\alpha_1(u_1+u_3)}}{[1-e^{-\alpha_1}-(1-e^{-\alpha_1 u_1})(1-e^{-\alpha_1 u_3})]^2} \\
&\quad \times \frac{\alpha_2(1-e^{-\alpha_2})e^{-\alpha_2(u_2+u_3)}}{[1-e^{-\alpha_2}-(1-e^{-\alpha_2 u_2})(1-e^{-\alpha_2 u_3})]^2} \times \frac{\alpha_3(1-e^{-\alpha_3})e^{-\alpha_3(u_4+u_5)}}{(1-e^{-\alpha_3}-(1-e^{-\alpha_3 u_4})(1-e^{-\alpha_3 u_5}))^2}
\end{aligned}
\tag{27}
$$

where

$$
u_4 = F(\varepsilon_2|\varepsilon_3) = \frac{\partial C_{23}(\varepsilon_2, \varepsilon_3)}{\partial \varepsilon_3} = \frac{(1-e^{-\alpha_2 u_2})e^{-\alpha_2 u_3}}{-(1-e^{-\alpha_2 u_2})(1-e^{-\alpha_2 u_3}) - e^{-\alpha_2} + 1}
$$

and

$$
u_5 = F(\varepsilon_1|\varepsilon_3) = \frac{\partial C_{13}(\varepsilon_1, \varepsilon_3)}{\partial \varepsilon_3} = \frac{(1-e^{-\alpha_1 u_1})e^{-\alpha_1 u_3}}{-(1-e^{-\alpha_1 u_1})(1-e^{-\alpha_1 u_3}) - e^{-\alpha_1} + 1}.
$$

8.2.3 Clayton-Frank Copula

The scatter plots of $(\varepsilon_1, \varepsilon_2)$, $(\varepsilon_1, \varepsilon_3)$, and $(\varepsilon_2, \varepsilon_3)$ in Figures 5-7 reveal different patterns of clusters of observations, suggesting that a finite mixture of copulae may provide a better fit than any single copula. Each mixture component is roughly corresponding to a cluster of observations, and the dependence structure may be different across mixture components. Finite mixture copulae have been used in Hu (2006) and Chen and Fan (2006).

Since Figures 5 and 6 show that $(\varepsilon_1, \varepsilon_2)$ and $(\varepsilon_1, \varepsilon_3)$ have lower tail dependence, we use the Clayton copula to model their dependence. The dependence structure of $(\varepsilon_2, \varepsilon_3)$ is not clear from Figure 5, so we use the Frank copula which is able to capture weak lower and upper tail dependence to model the dependence of $(\varepsilon_2, \varepsilon_3)$. Then using similar procedures as for the Clayton and Frank copulae, we derive the pdf function of the Minflex Laurent Clayton-Frank copula demand system

$$
\begin{aligned}
f(\varepsilon_1, \varepsilon_2, \varepsilon_3) &= f_1(\varepsilon_1) f_2(\varepsilon_2) f_3(\varepsilon_3) \times c_{1,3}(F_1(\varepsilon_1), F_3(\varepsilon_3)) \\
&\quad \times c_{2,3}(F_2(\varepsilon_2), F_3(\varepsilon_3)) \times c_{1,2|3}(F(\varepsilon_1|\varepsilon_3), F(\varepsilon_2|\varepsilon_3)) \\
&= f_1(\varepsilon_1) f_2(\varepsilon_2) f_3(\varepsilon_3) \times \frac{(1+\alpha_1)(u_1^{-\alpha_1} + u_2^{-\alpha_1} - 1)^{-\frac{1}{\alpha_1}-2}}{(u_1 u_2)^{\alpha_1+1}} \\
&\quad \times \frac{\alpha_2(1-e^{-\alpha_2})e^{-\alpha_2(u_2+u_3)}}{[1-e^{-\alpha_2}-(1-e^{-\alpha_2 u_2})(1-e^{-\alpha_2 u_3})]^2} \times \frac{(1+\alpha_3)(u_4^{-\alpha_3} + u_5^{-\alpha_3} - 1)^{-\frac{1}{\alpha_3}-2}}{(u_4 u_5)^{\alpha_3+1}}
\end{aligned}
\tag{28}
$$

where

$$u_4 = F(\varepsilon_2|\varepsilon_3) = \frac{\partial C_{23}(\varepsilon_2, \varepsilon_3)}{\partial \varepsilon_3} = \frac{(1 - e^{-\alpha_2 u_2})e^{-\alpha_2 u_3}}{-(1 - e^{-\alpha_2 u_2})(1 - e^{-\alpha_2 u_3}) - e^{-\alpha_2} + 1}$$

and

$$u_5 = F(\varepsilon_1|\varepsilon_3) = \frac{\partial C_{13}(\varepsilon_1, \varepsilon_3)}{\partial \varepsilon_3} = u_3^{-\alpha_1 - 1}(u_3^{-\alpha_1} + u_1^{-\alpha_1} - 1)^{-1/\alpha_1 - 1}.$$

We use the maximum likelihood method to estimate the Minflex Laurent demand system with each of the Clayton, Frank, and mixture of the Clayton and Frank copulae, based on the density functions in equations (26), (27), and (28), respectively. Since we relax the joint normality assumption in the disturbance terms and allow for nonlinear dependence across equations, there is no singularity in the covariance matrix of the residuals in the copula-based demand system. Thus, we do not need to delete any equations in our maximum likelihood estimation of the Minflex Laurent copula-based demand system. In this regard, we should note that Velasquez-Giraldo *et al.* (2018) use the maximum log-likelihood method to estimate a copula-based demand system by arbitrarily deleting one equation in the demand system. However, as Berndt and Savin (1975) has demonstrated when certain cross-equation restrictions are imposed, the parameter estimates obtained by trace minimization are not invariant to the equation deleted. In other words, the copula-based demand system estimates in Velasquez-Giraldo *et al.* (2018) are not invariant to the arbitrary deletion of one equation.

The results are presented in columns 3, 4, and 5 of Table 3. It seems that our copula-based modeling of the Minflex Laurent demand system has been fruitful. The copula parameters are statistically significant for all the copulae examined and the AIC values of the Minflex Laurent copula demand system are all lower than that under the traditional method of estimation (in the second column of Table 3). Most of the copula parameter estimates of the Clayton, Frank, and mixture of Clayton and Frank copulae are positive and statistically significant. Since we transformed $F_1(\varepsilon_1)$ to $1 - F_1(\varepsilon_1)$, the positive copula parameter estimates indicate that the dependence is negative for transaction balances, x_1, and OCDs (at commercial banks and thrift institutions), x_2, as well as for transaction balances and credit card transaction services, x_3, while the dependence is positive for OCDs and credit card transaction services.

The mixture copula has the highest log-likelihood function value and lowest AIC value, suggesting that the trivariate mixture copula-based Minflex Laurent demand system has the best goodness of fit. The mixture copula indicates that $(\varepsilon_1, \varepsilon_2)$ and $(\varepsilon_1, \varepsilon_3)$ exhibit lower tail dependence, while $(\varepsilon_2, \varepsilon_3)$ exhibits upper tail dependence. The lower tail dependence indicates that factors that cause negative shocks in one monetary asset tend to also cause negative shocks in the other assets, and vice versa. This phenomenon is similar to financial contagion — the spread of shocks (mostly on the downside) from one market (or country) to another.

It is to be noted that we also attempted to use the Joe (1993) copula. However, since the Joe copula does not permit lower tail dependence, but can only capture upper tail dependence, we experience computational problems in the estimation algorithm. With the Joe copula, the model fails to converge to any value. The fact that the Joe copula fails to converge can be interpreted as further evidence of misspecification that stems from using copulae that do not support negative dependence. As Trivedi and Zimmer (2007) argue, one might experience computational difficulties when using a misspecified copula.

9 Elasticities

The primary interest of policymakers is how the arguments of the underlying functions affect the quantities demanded. In our context, this is expressed in terms of income and price elasticities, as well as the elasticities of substitution.

In panel A of Table 5 we present the income elasticities, η_{iy}, for the three monetary goods evaluated at the mean of the data, based on the estimates of the mixture copula ML demand system. All the income elasticities are statistically significant. The income elasticity for transactions balances, η_{1y}, is 1.201 with a p-value of 0.000, suggesting that transactions balances is a luxury good. The income elasticities for credit card transaction services and OCDs are less than 1 — $\eta_{3y} = 0.938$ with a p-value of 0.000 and $\eta_{2y} = 0.645$ with a p-value of 0.000, respectively — suggesting that they are necessity goods. The literature has not reached a consensus on the magnitude of the income elasticity yet. The quantity theoretic money demand function implies a unitary income elasticity. Many empirical studies report income elasticities close to 1 (see, for example, Meltzer (1963), Feige (1964), Lucas (1988), and Teles and Zhou (2005)), but recent work reports both higher estimates (see Mulligan and Sala-i-Martin (1992)) as well as lower estimates (see Ball (2001)).

For comparison purposes, in panel B of Table 5, we also present the income elasticities based on the traditional estimation of the Minflex Laurent demand system. As can be seen, under traditional estimation, the income elasticity of credit card transaction services is also less than 1 ($\eta_{3y} = 0.866$ with a p-value of 0.000), but lower than that under the mixture copula Minflex Laurent demand system estimation. Panel B of Table 5 also shows that under the traditional Minflex Laurent demand system estimation, the income elasticity of transaction balances, η_{1y}, is 1.181 with a p-value of 0.000 and that of OCDs is $\eta_{2y} = 0.646$ with a p-value of 0.000, both very close to those based on the copula estimation.

We also present the own- and cross-price elasticities in Table 5. They reveal a pattern that is consistent with neoclassical consumer theory. That is, all own-price elasticities in panels A and B of Table 5 are negative (and statistically significant), consistent with the view that the demand for money is negatively related to the opportunity cost of holding money. Also, all the assets are own-price inelastic as $|\eta_{ii}| \leq 1$.

From the point of view of monetary policy, the elasticities of substitution among the monetary assets are of prime importance. If the credit card transaction services are substitutes to monetary assets, then it is necessary to include credit card transaction services into monetary aggregates. The currently popular simple sum approach to monetary aggregation requires that the components of the monetary aggregates are perfect substitutes to each other and that the elasticities of substitution between each other are very high (perhaps infinite). In Table 6 we show estimates of the Allen elasticities of substitution. We expect the three diagonal terms, representing the own-elasticities of substitution for the three assets, to be negative. This expectation is clearly achieved. Panel A shows that the Allen own-elasticities of substitution for the mixture copula-based Minflex Laurent demand system are $\sigma_{11}^{a} = -0.342$ with a p-value of 0.000, $\sigma_{22}^{a} = -0.559$ with a p-value of 0.000, and $\sigma_{33}^{a} = -0.867$ with a p-value of 0.000.

However, the Allen elasticity of substitution produces ambiguous results off-diagonal, and we use the asymmetrical Morishima elasticity of substitution to investigate the relation among the components of the M1A monetary aggregate. Based on the Morishima elasticities of substitution of the mixture copula-based Minflex Laurent demand system as shown in panel A of Table 6, all the assets are Morishima substitutes. Moreover, all the mean Morishima elasticities of substitution are less than 1, with the highest being $\sigma_{31}^{m} = 0.370$. We are interested in how changes in the user cost of credit card transaction services, p_3, affect the quantities demanded of the monetary assets, x_1 and x_2. As can be seen, $\sigma_{13}^{m} = 0.299$ and $\sigma_{23}^{m} = 0.167$, suggesting that a one percent increase in the user cost of credit card transaction services induces a 0.299 percent decrease in the relative demand for transaction balances, x_1/x_3, and a 0.167 percent decrease in the relative demand for OCDs at commercial banks and thrift institutions, x_2/x_3. It should also be noted that the Morishima elasticities of substitution between credit card transaction services and the monetary assets are larger under the mixture copula-based Minflex Laurent demand system estimation compared to the traditional Minfex Laurent demand system estimation (in panel B of Table 6). The positive elasticities of substitution between credit card transaction services and the monetary assets support the Divisia approach to monetary aggregation.

10 The Effects of Covid-19 on Payment Preferences

The sudden appearance of Covid-19 has been swiftly ravaging the United States and global economy. The unemployment rate in the United States shot up to 14.7 percent in April 2020, while personal consumption expenditures were almost 20 percent lower than at their peak in February 2020. This pandemic shock has reduced spending across all methods of payment, including cash, debit cards, and credit cards. At the meantime, the pandemic panic has led to an unprecedented demand for cash, and an accelerated adoption of cards and contactless payments. Although it is not surprising that the pandemic has led to a shift towards cash, the scale has been unprecedented. According to a Federal Reserve survey of consumers taken in May 2020 — see Kim *et al.* (2020) — during the pandemic, holdings of cash per person increased by 17 percent, from \$69 to \$81; and the amount of cash stored at home or elsewhere rose by nearly 90 percent, from \$257 to \$483. Even those consumers who favor the use of debit and credit cards, they were holding more cash in May than they were before the pandemic. The change in payment methods and demand for monetary services during the pandemic could be driven by the Covid-19 circumstances along with the changes in consumption patterns. It could also be driven by the changes in the opportunity costs of holding different monetary assets induced by the reduced federal funds rate and the Fed's unconventional monetary policy.

The literature on the change of payment patterns and demand for monetary services during the pandemic is growing fast and has attracted attention from both academia and central banks. Most studies use survey data and event studies. Coibion *et al.* (2020) studies how Covid-19 causally affects household spending and macroeconomic expectations using survey data. Kim *et al.* (2020) study how payment behavior has changed by the pandemic, given the dramatic increase in the demand for currency, along with anecdotal evidence of changing consumer payment practices during the pandemic. Similarly, Chen *et al.* (2020) survey the Canadian data and analyze the effects of the pandemic on the demand for cash and on the shift of payment methods in Canada, and find that cash in circulation in Canada grew sharply in March and April 2020. Garratt *et al.* (2020) uses Google searches data during the pandemic to demonstrate a shift in public interest from cash-related terms to digital payment options.

We analyze the demand for monetary services during the pandemic by looking at the dynamics of the time-varying Morishima elasticities of substitution. The shift in payment methods and the demand for monetary services reflects the rational re-allocation of economic activity by economic agents. Since our demand system estimators are obtained by using data starting from 2006 and the sample size of the Covid-19 pandemic period is relatively small, the demand system parameter estimates are likely to be dominated by the information before the Covid-19 crisis. Yet, such an assumption is plausible as consumer payment preferences are constrained by demographic and economic factors that are unlikely to change overnight, but the user costs of monetary services can change overnight in the financial markets. As Bullock (2020) and Brainard (2020) point out, there is still a significant number of people in the population, such as older people or people on lower incomes, who continue to use cash for face-to-face payments, due to limited access to banking and technology. However, if the pandemic persists and effective measures are put in place to overcome these barriers, with a longer sample period of data, the demand system parameter estimates will be more influenced by the information set and the demand for monetary services during the pandemic.

With this in mind, we investigate the stability of the time-varying Morishima elasticities of substitution over the sample period. In this regard, from the perspective of monetary policy, policy decisions based on targeting the money supply will be more effective if the Morishima elasticities of substitution are stable over time. In Figures 8-10, we plot the Morishima elasticities of substitution and also provide a comparison between those of the Minflex Laurent mixture copula demand system and those of the traditional Minflex Laurent demand system. As can be seen, the Morishima elasticities of substitution are always larger under the Minflex Laurent mixture copula estimation, than under the traditional estimation, suggesting a slightly higher instability of the asset demand functions.

We also find that under the Minflex Laurent mixture copula estimation, the Morishima elasticity of substitution between transactions balances and credit card transaction services has remained relatively stable during the pandemic, irrespective of changes in the user cost of transactions balances or credit card transaction services (see Figure 9). The differences between credit card transaction services and cash have been manifested during the Covid-19 pandemic and contribute to the relatively stable elasticity of substitution between credit card transaction services and transaction balances. Credit card transaction services can support social distancing through online payment and phone payment, while cash cannot; cash is a safe store of value during a crisis. The adoption of credit card transaction services is accelerated due to its social distancing properties. In this regard, Krueger *et al.* (2020) distinguish goods by their degree to which they can be consumed at home rather than in a social (and thus possibly contagious) context, and show that the decline in the demand for certain goods is simply due to rational reallocation of economic activity, such as shifts from partying together in bars to talking online, staying at home as opposed to congregating in restaurants. We observe similar shifts in the consumption of monetary services and credit card transaction services. The relatively stable Morishima elasticity of substitution between credit card transaction services and cash highlights the distinct social distancing features of credit card transaction services.

The other Morishima elasticites of substitution have declined significantly during the Covid-19 pandemic, exept for σ_{21}^m which increased (see the lower panel of Figure 8), suggesting that increases in the user cost of transaction balances increased the relative demand for OCDs (the x_2/x_1 ratio). The Morishima elasticity of substitution between transaction balances and OCDs when the user cost of OCDs changes, σ_{12}^m, as well as the Morishima elasticity of substitution between OCDs and credit card transaction services, irrespective of which user cost changes, fell significantly during the pandemic (see the first panel of Figure 8 and the two panels of Figure 10, respectively).

11 Conclusion

This paper contributes to the literature by investigating the demand for monetary assets when credit card transaction services enter the representative consumer's utility function. We use recent advances in microeconometrics and an econometric framework that allows the estimation of demand functions in a systems context, using a flexible functional form for the utility function based on the dual approach to demand system generation. We model the Minflex Laurent demand system, introduced by Barnett (1983), pay explicit attention to theoretical regularity, as suggested by Barnett *et al.* (1992), and relax the joint normality assumption of the disturbance terms of the demand system that has been used in most of the empirical monetary demand systems literature. In doing so, we use copula methods to capture the dependence of the disturbance terms across the monetary components. In particular, we express the joint distribution of the demand system error terms as a function of the marginals and copulae which are able to capture the dependence structure of the innovation terms across the demand system equations.

The empirical results, based on the Minflex Laurent demand system and mixture copula, show that the Morishima elasticities of substitution among transaction balances, OCDs at commercial banks and thrift institutions, and credit card transaction services are larger than those based on the estimation of the Minflex Laurent demand system under the joint normality of the errors assumption. The positive Morishima elasticities of substitution between credit card transaction services and traditional monetary assets suggest that credit card transaction services and traditional monetary assets are substitutes and that the credit card transaction services should be included in the monetary aggregates. Our results support Barnett *et al.* (2016), Barnett and Liu (2019), and Liu *et al.* (2020) who argue that much of the policy relevance of the Divisia monetary aggregates could be strengthened by the use of credit card-augmented Divisia monetary aggregates.

Finally, in terms of our framework, which is based on a strong link between neoclassical microeconomic theory and econometric implementation, the variation in the Morishima elasticities of substitution during the Covid-19 pandemic reflects the changes in preference structure for monetary services demand. The lower and stable Morishima elasticities of substitution during the Covid-19 pandemic indicate that the asset demand functions have become more stable and predictable, enhancing the Fed's ability to target key monetary aggregates to accommodate the demand for monetary services and affect general macroeconomic variations.

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Figure 1: Number of noncash payments

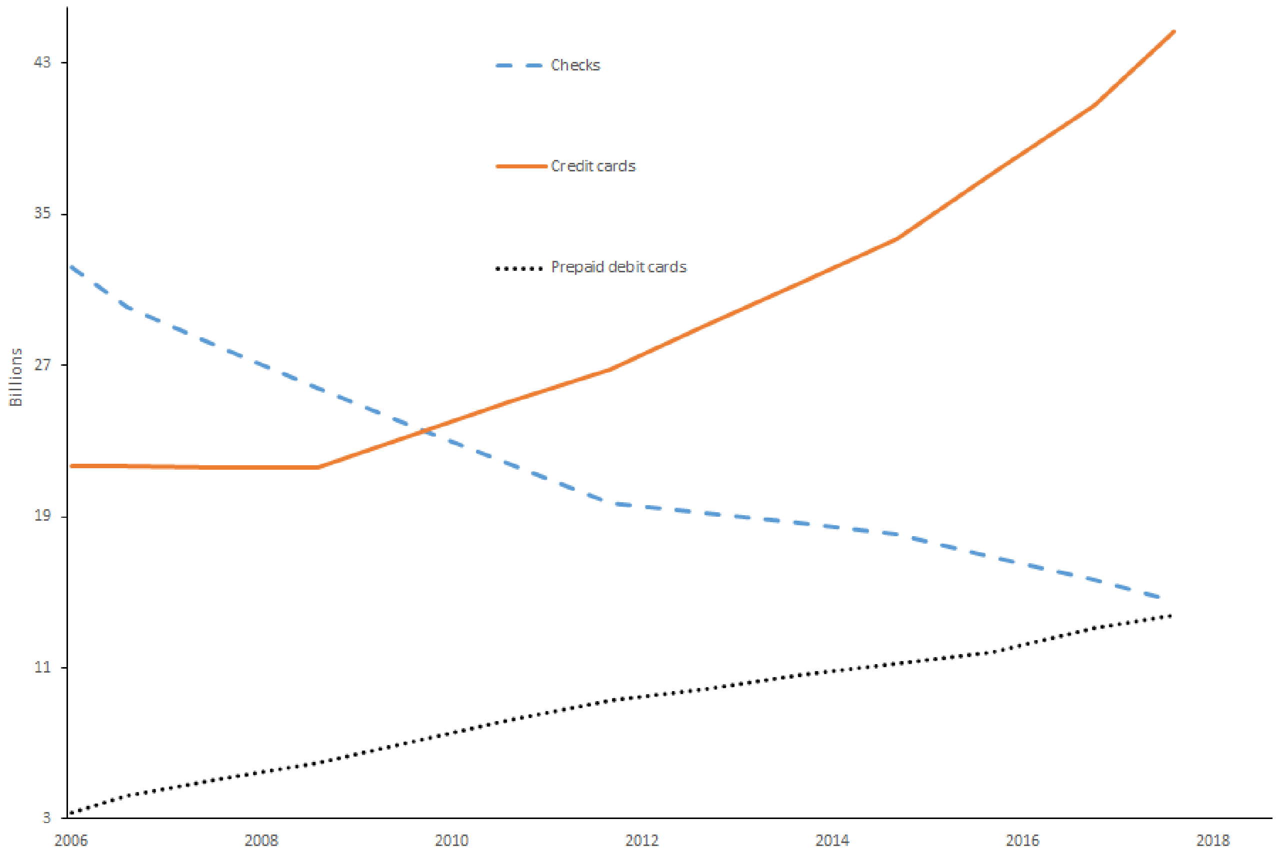


Figure 2: Value of credit card payments

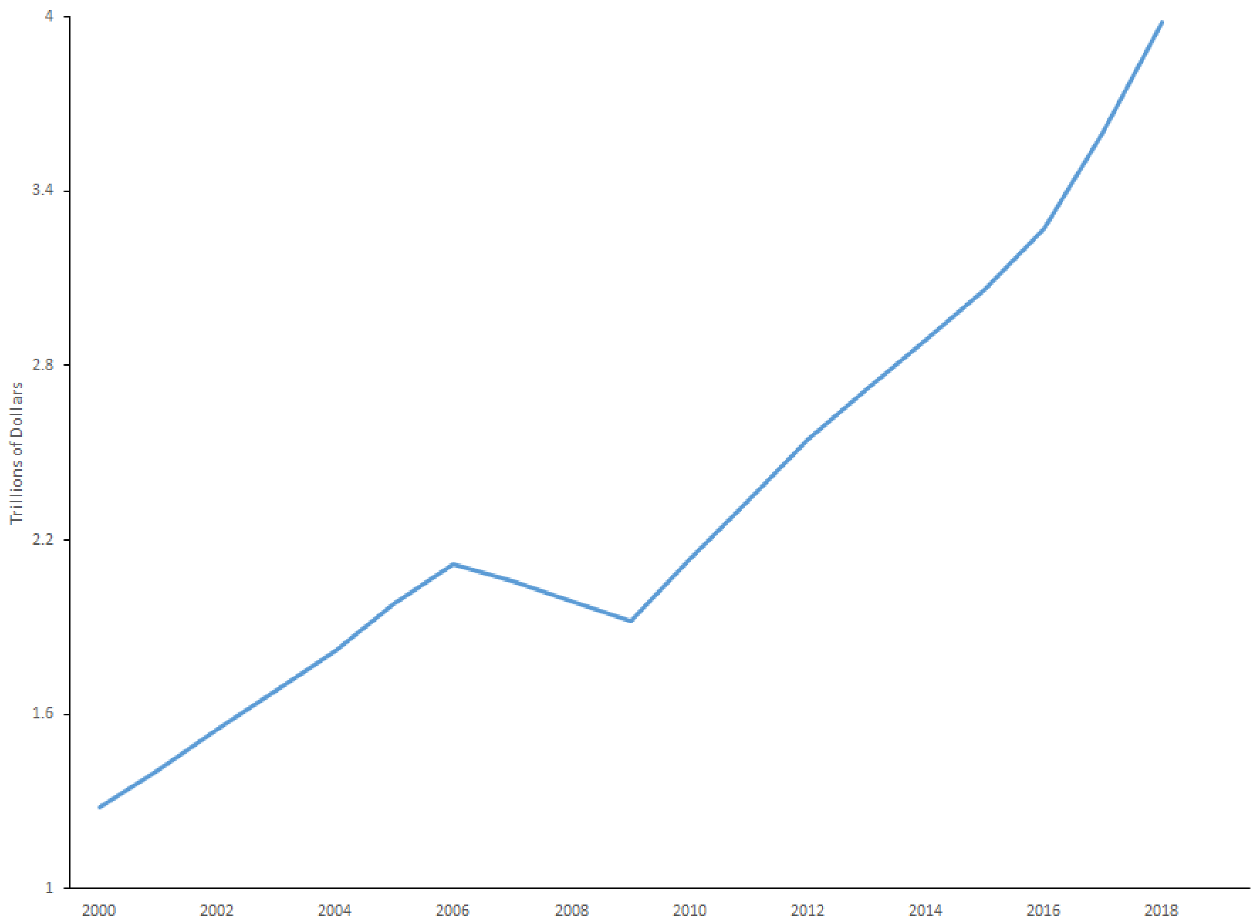


Figure 3: Log level of monetary aggregates

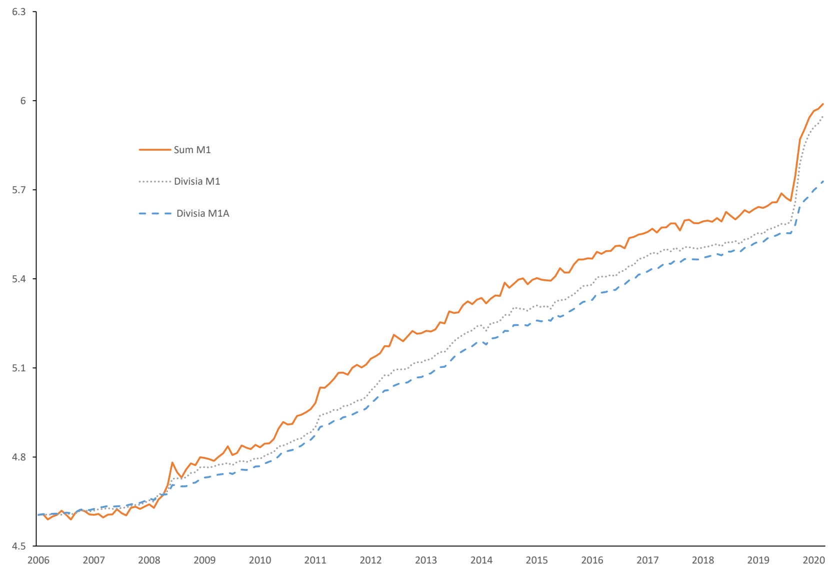


Figure 4: Year-over-year growth rates of monetary aggregates

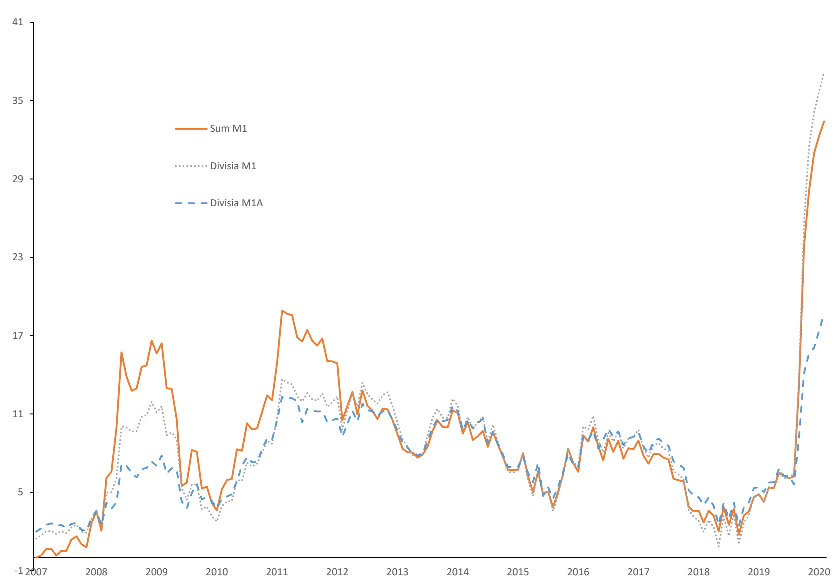


Table 1. Components and monetary aggregates in the United States

		CFS credit card-augmented Divisia monetary aggregates							
	Liquid asset	M1A	M2MA	MZMA	M2A	ALLA	M3A	M4A-	M4A
x_1	Transaction balances	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
	Currency								
	Travelers' checks								
	Demand deposits								
x_2	OCDs at commercial banks	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
	+ OCDs at thrifts institutions								
x_3	Credit card transaction services	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
x_4	Saving deposits at banks including MMDAs		$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
x_5	Saving deposits at thrifts including MMDAs		$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
x_6	Retail money-market funds		$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
x_7	Small time deposits at commercial banks				$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
x_8	Small time deposits at thrift institutions				$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
x_9	Institutional money-market funds			$\surd$		$\surd$	$\surd$	$\surd$	$\surd$
x_{10}	Large time deposits						$\surd$	$\surd$	$\surd$
x_{11}	Repurchase Agreements						$\surd$	$\surd$	$\surd$
x_{12}	Commercial paper							$\surd$	$\surd$
x_{13}	T-bills								$\surd$

Table 2. Utility maximization and weak separability tests

A. Utility maximization

Monetary aggregates	Utility maximization hypothesis	Number of GARP violations	Number of HARP violations
1. M1A:	$u(c, l, x_1, x_2, x_3)$	3	170
2. M2MA:	$u(c, l, x_1, x_2, x_3, x_4, x_5, x_6)$	3	170
3. MZMA:	$u(c, l, x_1, x_2, x_3, x_4, x_5, x_6, x_9)$	0	170
4. M2A:	$u(c, l, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$	0	170
5. MALL:	$u(c, l, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9)$	0	170
6. M3A:	$u(c, l, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11})$	0	170
7. M4A-:	$u(c, l, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12})$	1	170
8. M4A:	$u(c, l, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13})$	9	170

B. Weak separability

Monetary aggregates	Separability hypothesis	Number of GARP violations	Number of HARP violations
1. M1A:	$u(c, l, g(x_1, x_2, x_3))$	5	170
2. M2MA:	$u(c, l, g(x_1, x_2, x_3, x_4, x_5, x_6))$	108	170
3. MZMA:	$u(c, l, g(x_1, x_2, x_3, x_4, x_5, x_6, x_9))$	11	170
4. M2A:	$u(c, l, g(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8))$	104	170
5. MALL:	$u(c, l, g(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9))$	11	170
6. M3A:	$u(c, l, g(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}))$	18	170
7. M4A-:	$u(c, l, g(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}))$	45	170
8. M4A:	$u(c, l, g(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}))$	30	170

Note: Sample period, monthly 2006:7-2020:8. ($T = 170$).

Table 3. Minflex Laurent parameter estimates

Assets
1 = transaction balances
2 = OCDs at commercial banks and thrift institutions
3 = credit card transaction services

Parameter	Normal	Clayton	Frank	Mixture
δ_1	0.362 (0.000)	0.316 (0.012)	5.406 (0.000)	0.458 (0.000)
δ_2	0.000 (1.000)	0.000 (1.000)	11.120 (0.323)	0.000 (1.000)
δ_3	0.000 (1.000)	0.027 (0.992)	0.056 (0.989)	0.041 (0.877)
d_{11}	0.000 (1.000)	8.276 (0.591)	0.447 (0.678)	0.001 (0.999)
d_{12}	0.000 (0.045)	-0.228 (0.000)	-0.076 (0.000)	0.106 (0.000)
d_{13}	0.000 (0.000)	-0.000 (0.525)	0.000 (0.000)	-0.003 (0.000)
d_{22}	0.109 (0.070)	0.086 (0.829)	20.131 (0.000)	0.123 (0.540)
d_{23}	0.000 (0.998)	0.000 (0.000)	0.000 (0.922)	0.006 (0.584)
d_{33}	0.203 (0.010)	0.182 (0.899)	54.960 (0.000)	0.199 (0.519)
β_{12}	0.052 (0.026)	-0.000 (0.000)	0.000 (0.000)	0.001 (0.000)
β_{13}	-0.126 (0.000)	0.119 (0.000)	-6.524 (0.000)	0.121 (0.000)
β_{23}	0.000 (0.998)	0.007 (0.000)	-3.905 (0.000)	0.001 (0.584)
α_1		0.000 (0.920)	15.700 (0.000)	33.710 (0.000)
α_2		0.918 (0.000)	81.082 (0.000)	28.485 (0.000)
α_3		27.652 (0.000)	27.976 (0.000)	287.575 (0.000)
function value	927.762	1710.448	1104.146	1710.808
AIC value	-1921.523	-3384.897	-2172.292	-3385.615

Note: Sample period, monthly 2006:7-2020:8 (T = 170). Numbers in parentheses are p-values.

Table 4. Bivariate dependence

Series	Correlation	Kendall	Spearman
A. Full sample			
(ϵ_1, ϵ_2)	-0.757	-0.602	-0.824
(ϵ_1, ϵ_3)	0.024	-0.111	-0.040
(ϵ_2, ϵ_3)	-0.671	-0.287	-0.407
B. Non-recession period			
(ϵ_1, ϵ_2)	-0.833	-0.624	-0.842
(ϵ_1, ϵ_3)	0.087	-0.168	-0.077
(ϵ_2, ϵ_3)	-0.624	-0.209	-0.310
C. Recession period			
(ϵ_1, ϵ_2)	-0.197	-0.268	-0.277
(ϵ_1, ϵ_3)	-0.524	-0.489	-0.394
(ϵ_2, ϵ_3)	-0.732	-0.243	0.533

Note: Sample period, monthly 2006:7-2020:8 ($T = 170$).

Figure 5: Scatter plot of ϵ_1 and ϵ_2

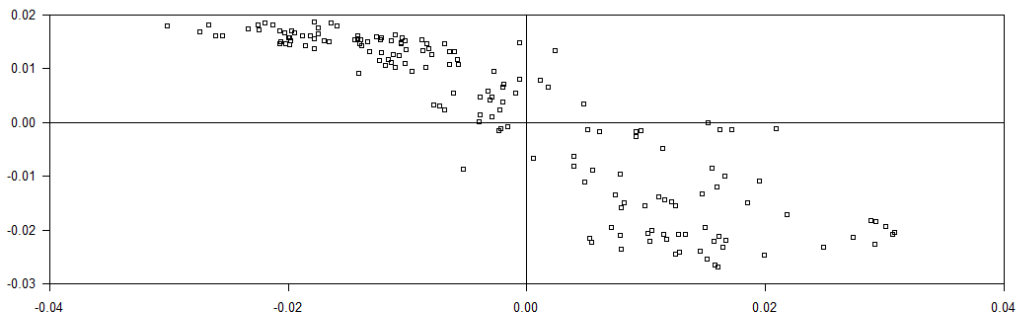


Figure 6: Scatter plot of ϵ_1 and ϵ_3

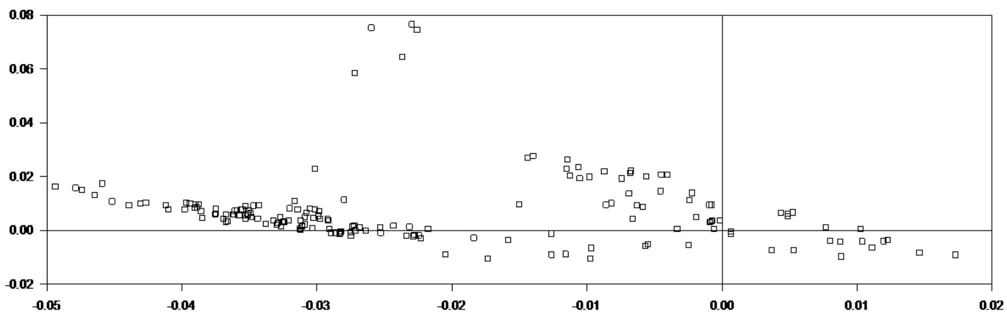


Figure 7: Scatter plot of ϵ_2 and ϵ_3

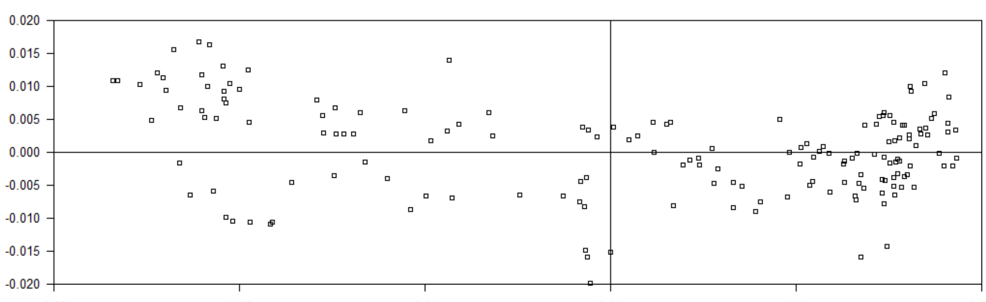


Table 5. Income and price elasticities at the mean

Assets
x_1 = transaction balances
x_2 = OCDs at commercial banks and thrift institutions
x_3 = credit card transaction services

A. Mixture copula Minflex Laurent demand system

	Income elasticities	Price elasticities		
Assets i	η_{iy}	η_{i1}	η_{i2}	η_{i3}
x_1	1.201 (0.000)	−0.772 (0.000)	−0.456 (0.000)	−0.281 (0.000)
x_2	0.645 (0.000)	−0.079 (0.000)	−0.258 (0.000)	−0.103 (0.000)
x_3	0.938 (0.000)	−0.257 (0.000)	−0.458 (0.000)	−0.449 (0.000)

B. Minflex Laurent demand system under the joint normality assumption

	Income elasticities	Price elasticities		
Assets i	η_{iy}	η_{i1}	η_{i2}	η_{i3}
x_1	1.181 (0.000)	−0.797 (0.000)	−0.561 (0.000)	−0.344 (0.000)
x_2	0.646 (0.000)	−0.080 (0.000)	−0.207 (0.000)	−0.091 (0.000)
x_3	0.866 (0.000)	−0.211 (0.000)	−0.369 (0.000)	−0.383 (0.000)

Note: Sample period, monthly data 2006:7-2020:8 ($T = 170$). Mean of the elasticities is reported in the table. Numbers in parentheses are p-values.

Table 6. Elasticities of substitution at the mean

Assets
x_1 = transaction balances
x_2 = OCDs at commercial banks and thrift institutions
x_3 = credit card transaction services

A. Mixture copula Minflex Laurent demand system

	Allen elasticities			Morishima elasticities		
Assets i	σ^a_{i1}	σ^a_{i2}	σ^a_{i3}	σ^m_{i1}	σ^m_{i2}	σ^m_{i3}
x_1	−0.342 (0.000)	0.286 (0.000)	0.396 (0.000)		0.180 (0.000)	0.299 (0.000)
x_2		−0.559 (0.000)	−0.061 (0.000)	0.312 (0.000)		0.167 (0.000)
x_3			−0.867 (0.000)	0.370 (0.000)	0.109 (0.000)	

B. Minflex Laurent demand system based on joint normality assumption

	Allen elasticities			Morishima elasticities		
Assets i	σ^a_{i1}	σ^a_{i2}	σ^a_{i3}	σ^m_{i1}	σ^m_{i2}	σ^m_{i3}
x_1	−0.197 (0.000)	0.207 (0.000)	0.335 (0.000)		0.128 (0.000)	0.260 (0.000)
x_2		−0.529 (0.000)	−0.088 (0.000)	0.231 (0.000)		0.156 (0.000)
x_3			−1.028 (0.000)	0.310 (0.000)	0.078 (0.000)	

Note: Sample period, monthly data 2006:7-2020:8 ($T = 170$). Mean of the elasticities is reported in the table. Numbers in parentheses are p-values.

Figure 8: Morishima elasticities of substitution between transaction balances and OCDs

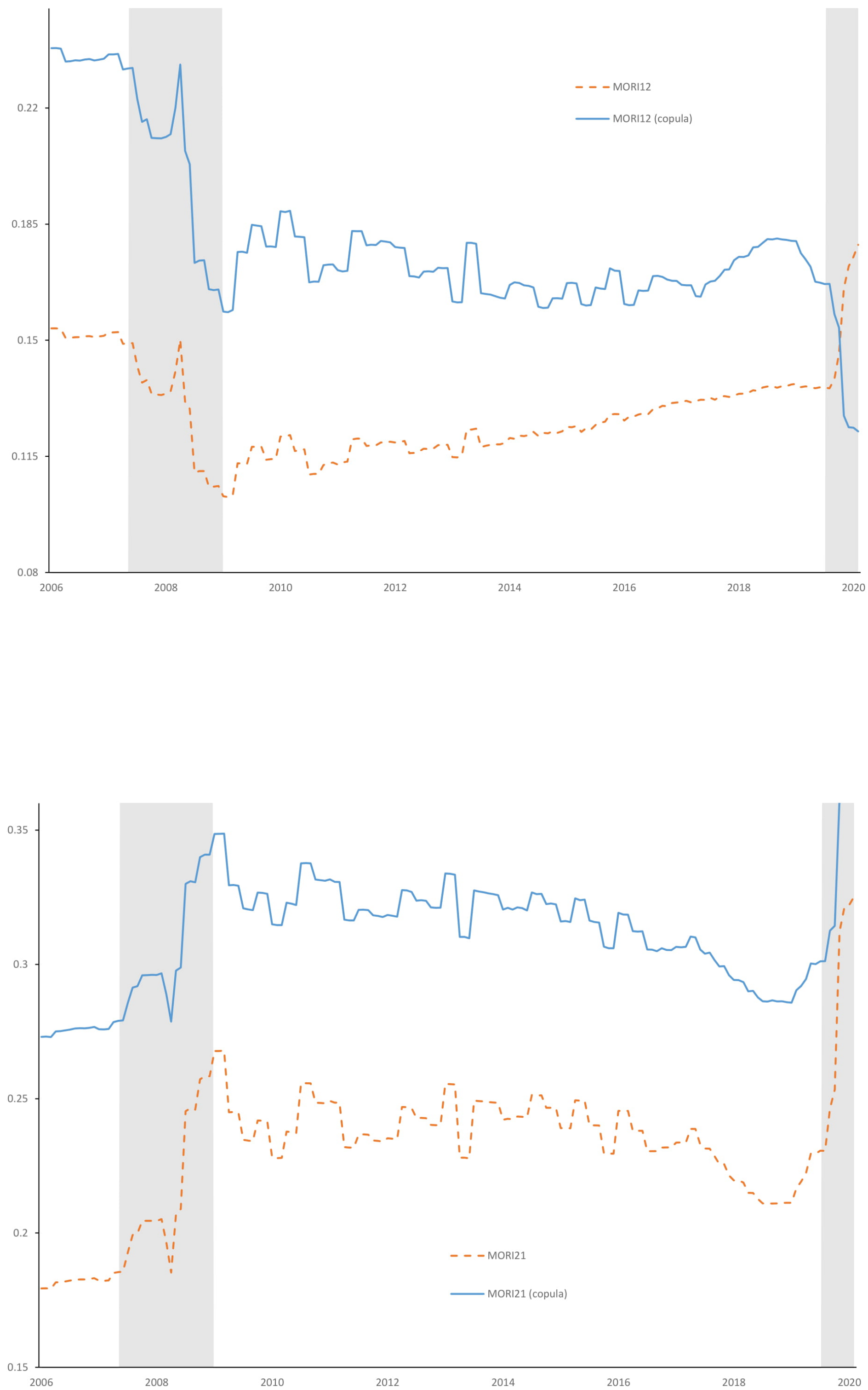
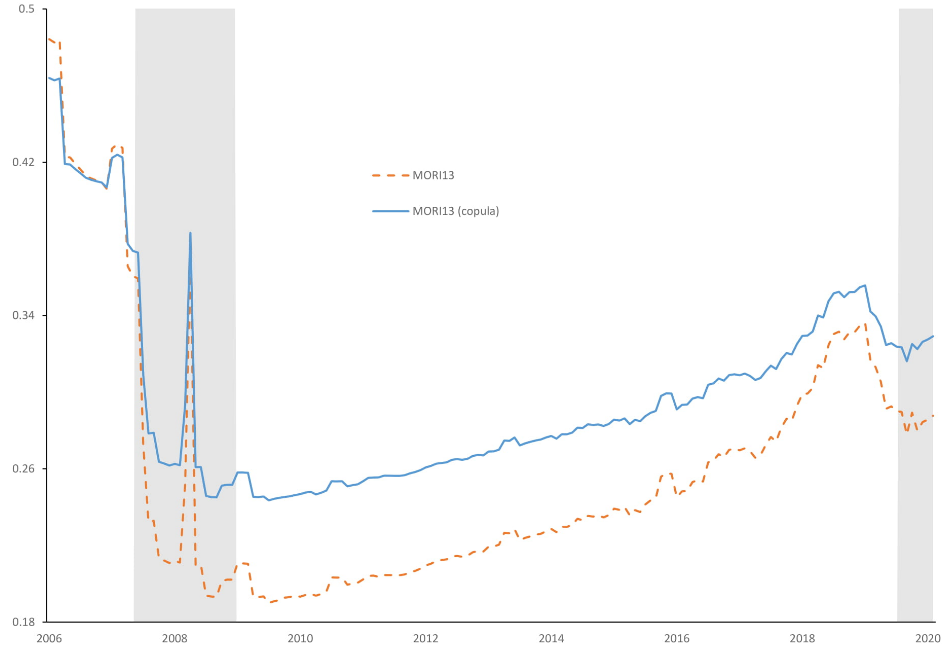


Figure 9: Morishima elasticities of substitution between transaction balances and credit card services



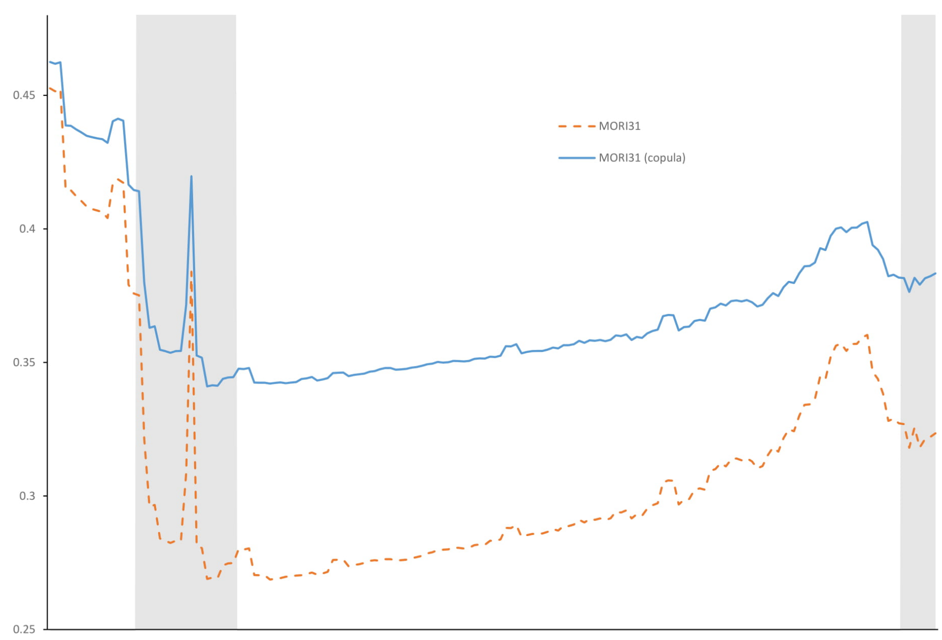


Figure 10: Morishima elasticities of substitution between OCDs and credit card transaction services