

Empty Names and Two-Valued Positive Free Logic

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1 Introduction

In order to tackle the problem of handling empty names in logic, Andrew Bacon (2013) takes on an approach based on positive free logic. He then provides a semantics for first-order fragments of languages containing non-referring terms (Bacon, 2013, p.16) and thereby provides a response to the second problem with empty names as well. With this semantics he is able to show that his version of positive free logic and semantics is sound while according to him the classical first order logic is unsound (Bacon, 2013, p.19).

Free Logic is a non-classical logic, free of existence assumptions for the singular and general terms. Its main point of departure from classical logic is allowing singular terms to denote objects outside of the domain of quantification, or not refer at all. Therefore it offers an appropriate way for distinguishing between atomic sentences that contain empty names and those that contain non-empty names instead (see Nolt, 2014).

Semantic views towards empty names can be divided into three categories, each being represented by one of the general approaches toward the semantics of free logic. According to Nolt (2014) these three common approaches can be presented as follows:¹

1. Negative semantics : All empty-termed atomic formulas are false.
2. Neutral semantics : All empty-termed non-existential atomic formulas are truth-valueless.
3. Positive semantics : Some non-existential empty-termed atomic formulas can be true.

Bacon argues for positive free logic as opposed to neutral and negative free logic. The semantics Bacon provides along with the logic he endorses at the end allows the sentence “there exists something that ‘Pegasus’ refers to” to be false and the sentence “there could be something that ‘Pegasus’ refers to” to be true simultaneously, and consequently allows some propositions containing non-referring terms to be true.

Bacon does not provide any arguments against a system in which some empty-termed propositions are true, some are truth-valueless, and the rest are false. In this essay, I invest

¹The first view is closer to that of Russell, and the third one is closer to Strawson’s view.

on the fact that not allowing any sentence to be truth-valueless is in contradiction with the premises that Bacon relies on in his rejection of negative free logic for empty names. Consequently, I argue, the closest consistent theory that is faithful to Bacon's premises is a system that allows all three truth-value conditions.

2 Bacon's theory of empty names

2.1 Arguments against negative and neutral free logic

Adopting negative or neutral free logic for modeling empty names results in serious problems in the model. Most importantly, in these logics the truth-value of sentences containing empty names is not dependent on the empty names themselves, but just on their emptiness. Hence by substituting an empty name with another one, the truth-value of the sentence will remain false, meaning that empty names have no semantic contribution to the sentences in which they appear, a statement that Bacon strongly disagrees with. For negative free logic, for example, this can be shown in the argument below with its unacceptable conclusion, the validity of which is the core of the problem ².

Argument 1.

1. All empty-termed atomic formulas are false.
2. If all members of a group of names are interchangeable in all atomic sentences without affecting the truth-value of them, they are not contributing to the semantic content.
3. Empty names are interchangeable in all atomic sentences without affecting the truth-value of the sentence. (From 1)
4. Empty names do not contribute to semantic content. (From 2 & 3)

Bacon assumes the second premise to be valid, tracing back the unacceptability of the conclusion to the invalidity of the first premise, i.e. the main premise of negative free logic.

To undermine the first premise, Bacon attacks (1), which is an assumption on which this premise relies.

- (1) Any property is existence-entailing.

What (1) amounts to is that the truth of a sentence in which a property is assigned to an empty name requires its referent's existence, and since the empty name in the sentence does not have an existent referent, the sentence is false.

²(an almost identical argument can be made for neutral free logic)

Bacon provides three sets of evidences against (1): ‘intensional transitive verbs’, ‘propositional attitudes’, and ‘counterfactual and fictional properties’.

The first class contains non-existence-entailing verbs like ‘draw’ and ‘worship’. The second class contains propositional attitudes a case of which is ‘believing’, and the last class contains sentences that assume the truth of the conditions in which a property can be attributed to an empty name. A list of some of the examples of such non-existence-entailing properties being attributed to empty names given by Bacons is presented below.

- The ancient Greeks worshipped Zeus.
- Had Doyle’s stories been true, Holmes would have been a private detective.
- Pegasus is a mythological horse-god.

All these properties are non-existence entailing, and yet intuitively seem to be able to form true sentences when assigned to empty names. They are therefore appropriate counterexamples, according to Bacon, to the general statement given in (1). He concludes from this that the first premise of the argument discussed above is not acceptable. Refuting the main premise of negative free logic leads to refuting negative semantics. Bacon gives a semantics according to which these sentences can be true.

2.2 Arguments in support of positive free logic

Beside rejecting negative and neutral free logic, Bacon also provides independent arguments in favor of positive free logic, in which some empty-termed propositions, i.e. those that attribute non existence-entailing properties to empty names, can be considered true.

So far we have seen how Bacon rejects negative free logic, and consequently accepts the following view toward the possible semantics of empty names;

An atomic sentence involving an empty name is true iff:

- 1) The property attributed to it is not existence-entailing
- 2) the property is true of the unique counterfactual referent of that empty name.

Otherwise it is false.

One can argue that an empty-termed proposition needs to rely on the referent of the name to be considered true, to which the properties of it are traceable. But a referent is by definition what empty names lack. Bacon’s answer to this problem is that while empty names

lack existing referents, they each have a “counterfactual referent”, to which the uses of those names are traced. Bacon argues that this counterfactual referent is uniquely determinable.

The first step towards establishing this claim is to show that ‘to refer’ is not existence entailing itself. Bacon provides two example-based arguments for this (Bacon, 2013, p.11);

1. There are non-referring names with referential properties;

He refers to the example ³ involving the vertices of a hypothetical triangle in a geometrical proof process, named ‘a’, ‘b’, and ‘c’. These are non-referring names that have properties. Therefore it is possible for non-referring names to have properties.

2. There are referring but not existential anaphoric pronouns;

For this, he provides the following example;

(2) If Phil had come, he would have made a dessert. It would have been pie.

This sentence contains a pronoun, ‘it’, which refers to a non-existing object, namely the dessert. These two facts allow him to conclude that ‘refer’ is not an existence entailing verb, therefore it is possible for empty names to refer while they have no referents. This guarantees the possibility of the semantic approach that Bacon follows.

From these examples Bacon conclude that ‘to refer’ itself is not existence entailing, and hence can be truly attributed to empty names. He also shows that free logic supports this claim about ‘refer’. Since empty names are not contained within the domain of quantification, both of the following two sentences can be stated without resulting in inconsistency:

1) $\neg\exists x(\text{'Pegasus' refers to } x)$.

2) ‘Pegasus’ refers to Pegasus.

These two are consistent because x in (1) cannot be substituted with an empty name. Hence it can be stated that empty names refer to nothing, while the sentence “‘Pegasus’ refers to Pegasus” is true.

By treating ‘refer’ as non existence-entailing, he can provide a determinate and unique counterfactual referent for empty names, based on the truth of the following counterfactual(Bacon, 2013, p.12):

(3) had p obtained, there would have been exactly one F , and the F would have been such that ‘a’ actually refers to it. where ‘a’ is an empty name, p is the condition under

³first presented by Frege in ‘On Sense and Reference’

which ‘a’ exists, and F is the counterfactual referent of ‘a’ .

Having provided this unique counterfactual referent for empty names, Bacon gains the ability to answer a worry raised by (Kripke, 1980) about the possible referents of empty names. Kripke’s statement is quoted below.

One cannot say of any possible person that he would have been Sherlock Holmes, had he existed. Several distinct possible people, and even actual ones such as Darwin or Jack the Ripper, might have performed the exploits of Holmes, but there is none of whom we can say that he would have been Holmes had he performed the exploits. For if so, which one? (Kripke (1980), p158)

In addressing this worry, Bacon’s answer is as follows;

A candidate referent of ‘Pegasus’ must be the subject of a baptism to which subsequent uses of the name ‘Pegasus’ can be traced back. To determine when something is a candidate ‘Pegasus’ referent we go to the closest world in which the Greek myths are true and look for the object satisfying the relevant features determined by the baptism.(Bacon (2013))

He then argues that what makes the referent determinate (or to some degree determinate) is that it is (at least to some extent) determinate in the closest possible world in which the statement “the Greek myths are true” is true. This answer is compelling but not clear enough. Later in this paper I discuss how and based on what unstated properties we can accept Bacon’s position on the uniqueness of the counterfactual referent of an empty name.

3 Potential Problems with Bacon’s Theory

3.1 The Premises of Bacon’s argument and their unforeseen consequences

Bacon’s semantic theory of empty names is based on taking the quality of being existence-entailing as the decisive factor in categorizing properties into two distinct sets. According to him properties in each category behave similar to each other semantically with regard to empty names. In other words, the argument is being premised on the assumption that atomic sentences that attribute one specific existence-entailing property to different empty

names, no matter what the property and empty names are, have the same truth value. I will henceforth refer to this premise as premise #1.

As I shall demonstrate, in Bacon's analysis it has not been taken into account whether there is a semantic relationship between the property and the empty name or not. In other words in Bacon's theory, 'whiteness' gives the same truth-value when assigned to 'Pegasus', 'Hulk', and 'Vulcan'. I will discuss later though, that these sentences carry different semantic properties that are not taken into account in this categorization.

In the following subsection I try to show how premise #1 is inconsistent with two different parts of Bacon's arguments.

3.1.1 Problem #1

According to what was said, Bacon assumes the following statement as a valid premise when he argues against negative free logic.

“If all members of a group of names ... are interchangeable in all sentence without affecting the truth-value of the sentence, they are not contributing to the semantic content”

On the other hand, according to premise #1, there is no difference between the truth values of any two atomic formulas that attribute an existence-entailing property to an empty name. Using an example, I try to show that these two positions are not compatible with each other and result in inconsistency.

I base my example on Bacon's example of the parabox. Parabox is a box that is both empty and non-empty, and given the paradoxical nature of its set of properties, it obviously never has an existing referent. Now let us expand this example by considering a set of such paradoxically defined boxes for each of which the only properties determined are a pair of contradicting existence-entailing properties. In this set, 'Parabox' is the empty and nonempty box, 'Parachromobox' is the blue and non-blue box, etc.

As we saw in the previous section, in Bacon's semantic theory the properties that are true of an empty name are those that are non-existence-entailing and are true of its counterfactual referent. Anything other than 'being empty', 'being nonempty', and 'being a box' is not true of the counterfactual referent of 'parabox' (or any other of the boxes in our example), and these three themselves fail because they are existence-entailing. Therefore according to Bacon's premises any atomic sentence about parabox (or any other member of the set of box

names we defined above) is false.⁴

Relying on the argument above, I can now claim that one result of Bacon's premises is the validity of the following argument;

Argument 2.

1. If all members of a group of names are interchangeable in all sentences without affecting the truth-value of the sentence, they are not contributing to the semantic content.
2. The group of the names of a set of boxes with paradoxical properties is such a group.
3. If all members of the group of names for the boxes with paradoxical properties are interchangeable in all sentence without affecting the truth-value of the sentence, they are not contributing to the semantic content. (From 1 & 2)
4. We can interchange any two members of the above mentioned group like 'Parabox' (the empty and non-empty box) and 'Parachromobox' (the blue and non-blue box) in any sentence without affecting the truth-value of the sentences.
5. Hence, these empty names do not contribute to the semantic content. (From 3 & 4)

The conclusion of this argument, however, contradicts another premise in favor of which Bacon argues. When arguing that empty names do contribute to the semantic content, Bacon specifically emphasizes that this is applicable to all empty names, including names with paradoxical properties.

It is therefore clear that the first premise of the above mentioned argument and premise#1, both endorsed by Bacon, contradict each other.

3.1.2 Problem #2

It follows from premise #1 that there is no semantic difference between different existence-entailing properties when they are applied to empty names. Such a semantic different does seem to exist, however, in some examples. This, if true, undermines premise #1.

Assume for example the following sentences, both attributing a color to an empty name;

⁴It could be said that anything that is implied by attributing these properties to a specific name, for instance 'being paradoxical', is also true about the referent, and if it is not existence entailing, it could be truly ascribed to it, but the point is that since the only difference between the members of that set is those two existence-entailing properties, then any such sentence attributing non-existence-entailing properties to them will have the same truth-value for all of them, therefore this will not affect the argument.

- Vulcan is red.⁵
- Pegasus is white.

At first glance through Bacon's semantic theory both these sentences seem to be false, and therefore probably semantically identical in all respects, since they are both ascribing an existence-entailing property to an empty name. Nevertheless, if they are all the same in semantic properties as premise #1 suggests, in Bacon's system there should be no such difference between these two counterfactuals either:

- Had we been in a Newtonian universe and had Mercury's orbit been as it actually is, Vulcan would exist, and would be red.
- Had Greek mythology been real, Pegasus would exist, and would be white.

The problem arises from the fact that these two sentences do differ. One can know that while the first two sentences are both false, in the second case as soon as one assumes that 'Pegasus' exists, the sentence will be true. Comparing this case with the case of 'Vulcan', we can see that assuming the existence of 'Vulcan' will have no effect on the truth value of the sentence. In other words the falseness of assigning a color to 'Vulcan' does not stem from the non-existence of the referent, while in the case of 'Pegasus' since the counterfactual referent of it carries the color, the falseness is caused only by the non-existence of the referent, and as soon as this problem is resolved, the sentence can be true.

Accepting this as a difference, the two first sentences can be considered as counterexamples for premise #1. It seems that being existence-entailing or non-existence-entailing is not the decisive factor in determining the semantic behavior of properties, even though it does play a role. There seems to be another distinction at play that makes 'Vulcan is red' and 'Holmes is red' different from 'Pegasus is white' and 'Hulk' is white.

3.2 How to resolve the inconsistencies

Given the inconsistencies discussed so far, two main questions need to be addressed. First, we want to know which premises should be abandoned to reach consistency, and second, we want to know whether reaching consistency is possible at all or not.

⁵I suppose here that no assumptions regarding the color of Vulcan have ever been made by anyone.

The first problem requires either premise #1 or the first premise of argument 2 to be abandoned or revised. Since the second problem is a problem with premise #1 itself, it seems that in the closest revised theory to that of Bacon premise #1 should be revised.

I argue later that the revision I suggest is not just one possible revision, but looking deeper in the requirements of Bacon's arguments in support of his theory reveals that they unanimously leads to this revised version.

3.2.1 Revising premise #1

The first problem suggested that the issue with premise #1 is that being or not being existence-entailing is not the only factor that results in differences in semantic properties of atomic sentences containing empty names. Therefore the revision should be such that it gives an account of those differences by giving a new categorization of properties.

To find the source of the problem in Bacon's account and give the new categorization such that it solves the problems, we need to concentrate on the difference between empty and non-empty names. As was mentioned in section 2.2, the ambiguity in Bacon's theory stems from the fact that he has not answered clearly the question of how the counterfactual referent he introduces for empty names is as determinate as the referent of non-empty names.

As I said before, Bacon supports a similar account of baptism like the one Kripke endorses, for empty names. According to him what results in the referent of an empty name to be uniquely and determinately recognizable is that it is a counterfactual referent that if existed, the name would actually refer to it. Therefore it will be the same as the empty name's referent in the closest possible world in which it exists, and since that possible world is determinate, the referent is determinate as well.

Bacon's answer, although it looks compelling as far as the uniqueness of the referent is concerned, does not capture the difference between any horse and the referent of 'Pegasus' for instance, or between any person carrying Holmes's properties, and the referent of Holmes. The answer I suggest to the question is that what makes them different is that the referent of 'Holmes' necessarily lacks some properties, or as one may say, the set of properties that can have determined truth-values if attributed to empty names are almost always a "proper" subset of the set of all possible properties.

This claim can be explained this way as well; for any empty name, there is a set of

properties that we don't know whether were true or false if the referent existed. This is what can be said about the redness of Vulcan, or Holmes Liking carrots.⁶

Accepting this, we can clearly answer Kripke's worry about the referent of empty names not being determinate. Since the referent of Holmes should necessarily lack a specific set of properties, then Darwin (or any other person) cannot be the referent of 'Holmes' because it does not lack those properties. Therefore for any empty name there will be a set of properties⁷ that just as Bacon claims are truly attributable to it, but also there is another set of properties that cannot lead to a determinate truth-value when attributed to the empty name, even in a counterfactual that presupposes the existence of the referent.

My new semantic theory that is based on this account of empty names is one according to which an atomic sentence involving an empty name is:

1) True iff the (non-existence-entailing) property ascribed to its counterfactual referent is among the properties determinately attributable to that name.⁸

2) Neither true nor false iff the property is among the set of non-determinately-attributable properties to the counterfactual referent of that name.

3) false otherwise.

It can now be said that the new theory is based on a new categorization of empty-termed atomic propositions. The dividing factor in this categorization is whether the property is determinately attributable to the empty name or not, as opposed to whether the property is existence-entailing or not, as Bacon suggested.

The difference between the new categorization and the old one shows up when non-determinately attributable properties are discussed. Attributing such properties to the empty name will result in a proposition that is neither true nor false, while in Bacon's system such propositions are either true or false.

Note that this is not simply a suggestion for resolving the problems raised in this paper, but as we have seen such a view is what is required for Bacon to be able to respond to Kripke appropriately as well. The only advantage of the unique referent suggested by Bacon over all

⁶I have supposed that this is something unknown according to the novels.

⁷The set will be empty in rare cases.

⁸It's not required to mention "non-existence-entailing" here because no existence entailing property can be truly attributable to empty names.

of Kripke's possible referents is that all properties other than the known properties should be necessarily undetermined about the referent of that empty name. As a result, even if all the properties that we know about the counterfactual referent of 'Holmes' are possessed by 'Darwin' too, 'Darwin' still cannot be the referent of 'Holmes' because there are many properties that are indeterminate about 'Holmes', but determinately true or false about 'Darwin'. The fact that the referent of an empty name should necessarily lack the unattributable properties is what makes the referent of the empty name unique and determinate.

3.2.2 Overcoming the two discussed problems under the new theory

So far I showed two problem in Bacon's system, traced them back to premise #1, and proposed a modified version of premise #1, demonstrating that its account of the counterfactual referent is compatible with Bacon's account of it. In the next step towards showing that it is an acceptable revised version of Bacon's theory, it should be shown that the problems with which Bacon's theory is faced cease to exist in the new theory.

The first problem involved empty names that do not contribute to the semantic content. In the revised version, for any empty name the set of existence-entailing and non-existence-entailing properties are both divided to determinately attributable and non-determinately attributable properties, with different truth-values when assigned to the empty name. Therefore for instance in my proposed version of the theory replacing 'Parabox' with 'Parachromobox' in the sentence 'Parabox is empty' changes the truth-value from false to neither true nor false, and the same is true for all empty names. Hence not contributing to the semantic content is no more a worry for empty names.

The second problem involved the semantic difference between cases of attributing the same property to different empty names. In the revised version, not only the problem is resolved, but also the theory gives an explanation of the differences between two such sentences. In the example discussed, 'whiteness' is a determinately attributable property to 'Pegasus', therefore the sentence "Pegasus is white" is false only because of the inexistence of the referent and the property being existence entailing. It follows that the counterfactual "Had Greek mythology been real, Pegasus would exist, and would be white." is true because it resolves the cause of the sentence's falseness. On the other hand "Vulcan is red" is neither true nor false. The inexistence of the referent is not the only cause of this truth-valuelessness. The property's

not being determinately attributable to Vulcan also plays a role. Thus the truth value will not change moving from that sentence to the counterfactual “had we been in a Newtonian universe and had Mercury’s orbit been as it is, Vulcan would exist, and would be red.”

It is noticeable that in the new version, the two sentences “Vulcan is red” and “Parabox was worshiped by an ancient people” are semantically similar, and so are counterfactuals like the one about Vulcan mentioned above and “Were Doyle’s stories factual, Holmes would like carrots”. This similarity could not be captured in Bacon’s semantic theory at all.

3.2.3 The problems with the revised theory

What I presented so far was a revised version of the semantic theory Bacon suggests for empty names, motivated by the need to resolve certain inconsistencies in Bacon’s initial theory. The suggested revision is compatible with Bacon’s premises in his arguments (more so compared to his own theory) without any of those problems. The next step is to revise the logic and semantics Bacon provides, but that is where the problem with the new account show up. Since in the new account some atomic empty-termed formulas are supposed to be truth-valueless, the options available for the logic and semantics are similar to those available for neutral free logic. Therefore, there will be two possible semantics, just like with neutral free logic: ordinary neutral semantics, and a semantics based on supervaluations.

None of these two options is easy to adopt as an appropriate semantics for this logic. If we choose ordinary neutral semantics, we will be dealing with a three-valued logic, in which it would be difficult to have tautologies while considering counterfactuals with false antecedents and truth-valueless consequents as truth-valueless.⁹

4 Conclusion

I tried to show that for Bacon’s semantic theory to be consistent and acceptable, one premise in his argument should change. The premise is the one according to which no difference is postulated between properties that are determinately attributable to an empty name and those that are not. The result is that if all other premises of Bacon’s argument are considered to be valid, then some empty-termed atomic formulas should be considered truth-valueless.

⁹These kinds of problems may lead one to build the semantics on supervaluations, which has its own issues. See Sorensen (2013) for a short introduction on this topic.

This account revises the categorization of properties, changing it to a system of categorization that also takes into consideration the determinacy of attributing a property to each name.

My work is only trying to show an inconsistency in Bacon's theory, not to support any specific semantic theory for empty names. I have tried to find the unasserted consequences of Bacon's thesis, and give the closest consistent theory to his theory.

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