

# Analyticity and Logicality in Semantic Constraints

Samara Burns

## 1 Introduction

Logic is often said to be “formal” in some sense. This is generally taken to mean that logical inferences, and therefore valid logical arguments, are those where the conclusion follows based on the *form* of the constituent sentences alone. Catarina Dutilh Novaes (2011) and John MacFarlane (2000) cite this notion as one of the ways we can think about the formality of logic. More specifically, this notion of formality is often taken to mean that logic is formal because it abstracts from the particular attributes and identities of the objects involved in the arguments. This is intuitive to most people who have taken an introductory logic class; logically valid arguments do not depend on facts about the objects referred to by the terms in the argument. This is in fact the position taken by many philosophers of logic (Sher (1991, 2008), Tarski (1986), and MacFarlane (2000)). Gil Sagi (2014) refers to this principle as one of the tenets of formality; I will refer to this principle throughout as F1.

**F1:** The logical validity of an argument is determined by its form.

It is often taken for granted that the logical form of an argument is determined by the logical language involved, and the arrangement of all terms in the argument. Sagi (2014) refers to this postulate as another tenet of formality; I will refer to this principle as F2.

**F2:** The logical form of an argument is determined by the logical language and the arrangement of all terms in the argument.

In logic, a language is generally divided into parts: the logical ( $\wedge, \vee, \neg, \forall, \dots$ ), and the non-logical (predicates, constants, punctuation...). This distinction is essential to determining logical consequence; Alfred Tarski (1936) defines logical consequence in a way that depends on this very distinction. Suppose we wish to know if some sentence  $K$  follows from a set of sentences  $L$ . On the Tarskian account, we can determine the consequence relation by replacing all non-logical symbols in the relevant sentences with variables (replacing like for like). The result is a set of what Tarski calls *sentential functions*. A model of the sentential functions consists of an arbitrary sequence of objects that satisfies the functions. If every model of the set  $L$  is also a model of  $K$ , then  $K$  follows logically from  $L$ .

Tarski's definition of consequence was first published in 1936. At the time, he did not give a demarcation of the logical and non logical terms. He was, in fact, skeptical about the possibility of such a demarcation (Tarski, 1936, 419). However, in 1966 he gave a lecture on permutation invariance - a mathematically precise way to determine the logical terms.<sup>1</sup> His discovery went relatively unknown until its publication in 1986. Permutation

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<sup>1</sup>Permutation invariance is taken to define the logical parts of a language, but is also used to demarcate logic as a whole. That is, it serves as a foundation for both F1 and F2. See MacFarlane (2000) for more details.

invariance determines the logical parts of a language by “swapping” objects in the domain of an interpretation for other objects. Those parts of the language that are unaffected by such a swap are considered the logical notions.

Given its close relationship to both F1 and F2, permutation invariance, if successful, would provide a precise explication of the formality of logic, and would give grounding to the Tarskian notion of consequence. However, several objections have been leveled against permutation invariance, and I argue below that it does not meet the requirements for a principled demarcation.<sup>2</sup>

This paper proceeds in the following way. I will first introduce the notion of permutation invariance, and provide several arguments as to why it fails to give a principled demarcation of the logical terms. Having motivated the problems with creating a principled demarcation in this way, I will introduce Sagi’s theory of semantic constraints. The theory of semantic constraints bypasses these demarcation issues by adopting F1 as an account of formality without adopting a demarcation of the logical terms.

In the final section I argue that Sagi’s account of semantic constraints does not capture logicity in a manner consistent with F1. I show how this is a consequence of reducing the form of an argument to the form of a language. Semantic constraints resemble a system for the analysis of analytic statements rather than logical arguments.

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<sup>2</sup>A principled demarcation should not simply be a list of *what we think* the logical operators should be; it should be substantive in some sense.

## 2 Permutation Invariance

An account of the permutation invariance criterion serves two purposes. First, it offers motivation for Sagi’s theory of semantic constraints by showing the problems that arise when attempting a principled demarcation of the logical terms. Second, permutation invariance is a popular way of characterizing formality in terms of form; it gives a precise method that corresponds to the intuitive notion that logic abstracts from the attributes of particular objects. This will be important in my discussion of semantic constraints in section 4.

Permutation invariance is the idea that the logical operators are those that are invariant under arbitrary permutations of the domain. Alfred Tarski was perhaps the first to apply the method of permutation invariance to determine the logical terms (what he called “notions”) (Tarski, 1986, 145). This method was used successfully in Felix Klein’s foundational work on the *Erlangen program*. Klein used the method to determine which properties are unique to different types of geometry. For instance, the notion of proportion is invariant under Euclidean transformations, and betweenness is invariant under affine transformations. Tarski theorized that the logical notions are those that are invariant under the widest class of possible permutations - arbitrary permutations of the domain onto itself (Tarski, 1986, 149). In this case, the notions that are captured are general in a desirable way, as an arbitrary “swapping” of objects in the domain ensures that the logical notions are not affected by the attributes of the objects.

In order for the permutation to work, it must abide by certain conditions. For instance, if an object A is permuted with an object B, then all instances of A in the interpretation must be swapped with B. Objects are also subject to metaphysical conditions; objects cannot

multiply or disappear.

Now consider the quantifier  $\forall$ . If a universally quantified sentence is true in some interpretation, then it is true regardless of what the objects in the domain of the interpretation are. We can therefore consider it a logical operator. In contrast, a quantifier such as “all humans” is sensitive to the properties of the objects in the domain. Generally speaking, the logical operators captured by the method are identity, existential and universal generalization, and the truth functional operators.<sup>3</sup> More surprisingly, quantifiers such as the cardinality quantifiers (“there are exactly  $k$  things such that...”), the well-ordering quantifier (“is a well-ordering”), the uncountability quantifier (“there are uncountably many”) and the relational and monadic “most” are also logical (Sher, 2008, 304). Because the criterion is meant to be substantive, it is unsurprising that some operators are considered logical that are not usually considered to be.

One might object to the criterion based on the inclusion of operators that do not seem to be logical. It might be the case that permutation invariance “lets in” too much mathematics, thereby creating a demarcation that is not truly logical (Dutilh Novaes, 2013, 7–8). Further objections have been raised based on the undergeneration of the theory. Dutilh Novaes (2013) argues that permutation invariance is insufficient to deal with the variety of logics seen in contemporary logical practice (9). For instance, the criterion is not amenable to modal logics. If demarcations are meant to reflect current practice in logic, then permutation invariance fails as a criterion.

Whether or not these specific objections are persuasive, permutation invariance faces a

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<sup>3</sup>Considered as objectual operators (such as union and intersection) they are included. However, without alterations to the method, they do not turn out as logical when considered as propositional operators. See MacFarlane (2000) and Sher (2008) for a further discussion.

problem. At some point the method stops being a substantive criterion and begins to be a reflection of antecedent intuitions about logicality. Those who argue in favour of permutation invariance often alter the method the results are not initially satisfactory - one can strengthen the method to let in fewer operators, or open it up to include more operators (MacFarlane (2000), Bonnay (2008)). This has led to disparity in the literature on permutation invariance as to which operators are logical.<sup>4</sup> However, as soon as one begins altering the criterion, it appears as if the alterations are doing the work rather than the criterion itself. Given these problems with permutation invariance, I argue that more work needs to be done before we can consider it a principled demarcation.

### 3 Semantic Constraints

I have given motivation for rejecting the notion of a principled demarcation by showing how our best attempts fail (or at least fail to be *principled*). Sagi (2014) argues that we can keep F1 as a tenet of formality without relying on F2 as is usually done. Her theory makes use of semantic constraints. These constraints are used to limit the class of allowable models by fixing the meanings of terms in the metalanguage. However, instead of following the usual logical/non-logical distinction, semantic constraints open up the possibility of a spectrum of logicality where terms are partially or completely fixed in the domain.

Consider a language  $L$ . An interpretation  $M = \langle D, I \rangle$ , where  $D$  is a (non-empty) domain and  $I$  is an interpretation function for  $L$ . I will use  $T$  and  $F$  as representative of the truth

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<sup>4</sup>For example, Sher (2008) rejects that any non-classical logic is logical, based on the permutation criterion. By contrast, MacFarlane (2000) accepts that many-valued logics and the modal logic S5 (though no others) are logical, based on the same criterion.

values.

Semantic constraints are already used in traditional logical semantics to fix the meaning of logical terms in the metalanguage. For instance, we may wish to fix the meaning of  $\wedge$  to ensure that a conjunction is only true when both of its conjuncts are true. So we will have, in the metalanguage, a sentence of the form,

$$\mathbf{C1: } I(\phi \wedge \psi) = T \text{ iff } I(\phi) = T \text{ and } I(\psi) = T.$$

Usually, constraints will be given for all the logical terms in the language. Semantic constraints are consistent with this traditional semantics. If one wishes to model arguments in classical logic, then she can choose to fully fix the meanings of the usual truth-functional connectives, the quantifiers and no more. However, since Sagi's main goal is to provide a semantics that does not adhere to the dichotomy between logical and non-logical, semantic constraints are not limited to full constraints.

As an example, consider an inference from “This apple is red” to “this apple is coloured”. On a classical account, this argument is not valid. However, under a theory of semantic constraints we can fix the meanings of “red” and “coloured” so that the former is a subset of a latter. Doing so would ensure that all red objects in the domain are also coloured (but not that all coloured objects are red). This is an example of a partial constraint. In the metalanguage, it would look like:

$$\mathbf{C2: } I(\text{red}) \subseteq I(\text{coloured}).$$

It is evident how Sagi's theory opens up the boundaries of logicity. Even though the validity of arguments is still dependent upon the form, it is possible to create a larger class of logically valid arguments. Specific sets of constraints may be beneficial for specific tasks. One could use a set of constraints similar to C2 to investigate the logical properties of colour,

or a set of constraints about height to investigate the logical properties of height.

The initial formulation of F2 is insufficient for semantic constraints. Instead, Sagi formulates it as such:

**F2\***: The logical form of an argument in a language  $L$  is determined by a set of semantic constraints for  $L$  and the arrangement of all terms in the argument.

Note that the logical validity of an argument is determined by the semantic constraints and the arrangement of terms in the argument, *rather than* by the arrangement of logical and non-logical terms.

A few things should be noted about semantic constraints. First, Sagi concedes that there is a possibility for there to be a *correct* set of semantic constraints. One could argue for a correct set of constraints in the same way that they could argue for a correct demarcation of logical constants. Second, semantic constraints are meant to *represent* the form of the language. Although this is ambiguous, one possible interpretation is that the form of the language is already there in some sense, and the semantic constraints are merely a codification of that form, rather than a definition of the form. How one is meant to determine the form of the language, and hence the set of semantic constraints, requires further inquiry.

## 4 Analyticity and Logicality

A theory of semantic constraints allows us to accept the notion of formality as pertaining to form without having to rely on a principled demarcation to do so. In this section I raise two

possible objections to the theory. First, I argue that semantic constraints do not capture the meaning of “form” as it is usually used. The theory reduces the form of an argument to the form of a language, which goes against traditional usage of “logical form.” My second objection is that semantic constraints are more closely aligned with the notion of analyticity than logical form.

Let us return to the notion of permutation invariance. Permutation invariance captures the notion of formality in logic by showing how logical terms abstract from the attributes of objects in the domain. Logical consequence on Sagi’s account, however, seems to take the attributes of the objects into consideration by allowing us to model arguments about colour and other properties. This problem seems to arise by conflating the logical form of an argument with the form of a language.

As mentioned before, Sagi notes how semantic constraints are representative of the form of a language. Despite the importance of this notion to her theory, it is merely mentioned in passing. I argue that this should be promoted to the status of a postulate. Since semantic constraints are dependent on the form of the language, and the form of an argument is dependent upon the semantic constraints, the form of a language plays a large role in determining logical validity. This postulate can be formulated as follows:

**F3:** A set of semantic constraints for a language  $L$  is determined by the form of  $L$ .

Note how this also leaves open the possibility of more than one set of constraints for  $L$  (in line with  $F2^*$ ). I take this to mean that the form of a language  $L$  can be represented in many ways (through various sets of semantic constraints) and not that  $L$  has more than one

form. A brief look at this principle raises several questions. How do we determine the form of a language? How do we ensure that the form is adequately represented by the semantic constraints? It seems as if logical form, in the absence of a method for determining the form of a language, is simply what we wish it to be.

Furthermore, suppose that we do have a way to determine the form of a language, and have a set of constraints representative of that form. I argue that this is not sufficient to capture F1. Keeping in mind the spirit of F1 (abstraction from the attributes of objects), take the conjunction of  $F2^*$  and  $F3$ . We get the following principle of formality:

**F4:** The logical form of an argument in a language L is determined by the form of L and the arrangement of all terms in the argument.

This appears to be where the problem occurs. When talking about the form of an *argument*, it is not the case that we are talking about the form of a *language*. Abstraction from attributes of objects usually includes abstraction from the relationships between properties or names in the language. This is why permutation invariance does not provide a demarcation that includes relationships between terms. Semantic constraints appear to have reduced the form of an argument to analytic properties of language. For this reason, I take it that many proponents of  $F1$  would not accept  $F4$ .

Instead of a conception of logic as formal, Sagi seems to have given an account of logic as analytic. That is, logical inferences (or arguments) are those that are knowable based on the meanings (or definitions) of the words involved. The examples Sagi uses for constraints are traditional examples of analytic statements. For example, the constraints:

**C2:**  $I(\text{red}) \subseteq I(\text{coloured})$ , and

**C3:**  $I(\text{bachelor}) \subseteq I(\text{unmarried})$ ,

are obviously analytic. The meaning of the word bachelor is an unmarried male, and red is defined as a colour. Since analytic statements are those that are knowable based on meanings or definitions, it could be that these types of statements represent the form of a language. Sagi also notes how semantic constraints are similar to Rudolf Carnap's theory of meaning postulates. Carnap proposed meaning postulates as a way to logically analyze analytic statements. According to Carnap, the difference between logically valid statements and analytic statements is the fact that logically valid statements are true without any reference to linguistic meanings (Carnap, 1952, 65–66). Analytic statements, although they require no empirical facts, have their validity based in logic and definitions. Carnap's aim was to provide a logical analysis of analytic statements, despite Quine's locution that no such analysis could be given (Carnap, 1952, 66).

Let me give a brief example. Consider a language  $L$  with primitive descriptive predicates  $R$ ,  $C$ ,  $B$ , and  $M$ , which describe our ordinary language *red*, *coloured*, *bachelor*, and *married*, respectively. Given the usual demarcation of logical terms, we can create meaning postulates of the following form:

**P1:**  $\forall x(B(x) \supset M(x))$

**P2:**  $\forall x(R(x) \supset C(x))$

According to Carnap, it is based on the understanding of terms that we decide which postulates to use. So there are several possibilities for a postulate that defines the relationship between two descriptive predicates. As such, there are several ways of capturing the “form” of the language.

Sagi's account is distinct from Carnap's insofar as the constraints are in the metalanguage. Meaning postulates are given in the object language and make free use of the logical terms. Carnap's theory is also meant as an analysis of language, whereas the theory of semantic constraints is used to determine the formality of logic. It is unclear how a tool for the analysis of analytic statements can be doubly used as a tool for the analysis of logical form.

What are semantic constraints meant to represent? Based on the professed similarity between meaning postulates and semantic constraints, it is natural to assume that the definitions or meanings of terms constitute the form of a language. If this is the case, then the notion of logical validity can be reduced to analyticity. Based on explications of  $F1$  such as permutation invariance, it is evident that analyticity is not what is meant by the tenet, and so Sagi's theory cannot adequately represent  $F1$ .

Of course there is one objection: The current way of interpreting  $F1$  is *wrong*.  $F1$  relies on an assumption that is unfounded (that there is a principled demarcation of logical terms). In order to avoid this assumption, we are led to the conclusion that the form of an argument is determined by the form of the language itself. Arguments based on the usual interpretation of  $F1$  fail because the usual interpretation is based on unfounded assumptions. I argue that if this were the case, then semantic constraints are still not a viable way of determining the formality of logic, as analyticity does not appear to be a (serious) candidate for the formality of logic.

Consider another disanalogy between meaning postulates and semantic constraints. Where semantic constraints are used as a semantics for logic, Carnap uses logic as a *tool* to construct an analysis of analytic statements. To use logic as a tool for such an analysis leads to problems if we take logic itself to be analytic; to claim that logic is analytic is to create

circular argumentation. The mere definition of analyticity would have to change - for to say that an analytic statement is one that is true based only on logic and definitions, is to say that a logically valid statement is one that is true based only on logic (since the definitions of a language have been subsumed by the logic). This seems wrong.

If one wishes to argue this route, then she will have to show why it is acceptable to reduce F1 to analyticity. She will then have to argue against the fact that logically valid statements and analytic statements are distinct, in some sense. On a pragmatic level, she must argue that her reasons for conflating the two have more weight than the reasons for keeping the distinction between logicity and analyticity (such as its historical importance). So far, semantic constraints do not seem to be a promising candidate for an explication of logical form.

## 5 Conclusion

I have shown that semantic constraints do not offer a promising account of the formality in logic. Due to the similarity with meaning postulates, it is likely that semantic constraints offer an analysis of language moreso than logic. Because the theory of semantic constraints conflates the notion of analyticity with logicity, the theory fails to capture the intuitive notion of formal as pertaining to logical form. Whether or not there is some alternative semantics that avoids assuming F2, but does not conflate logicity with analyticity, is still an open question.

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